CHAPTER 9: INTRODUCTION TO ALGEBRA

Chapter Objectives

By the end of this chapter, students should be able to:

✓ Interpret different meaning of variables.
✓ Evaluate algebraic expressions.
✓ Identify properties of algebra: commutative, associative, identity, and inverse.
✓ Solve multi-step equations.

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SECTION 9.1 Properties of Algebra

A. Identity Properties of Addition and Multiplication

There are two properties of algebra that you are probably very familiar with.

What happens when you add zero to a number? The sum of any number and zero is the number itself. We call this the **Identity Property of Addition**. Zero is called the additive identity. For example,

\[
13 + 0 = 13 \\
0 + 9 = 9
\]

What happens when you multiply a number by one? Multiplying a number by one does not change its value. We call this the **Identity Property of Multiplication**, and 1 is called the multiplicative identity. For example,

\[
43 \cdot 1 = 43 \\
1 \cdot 7 = 7
\]

**YOU TRY:**

What identity property is being used?

| a) \(125 \cdot 1 = 125\) | b) \(0 + 49 = 49\) |

B. The Commutative Properties of Addition and Multiplication

You may encounter daily routines in which the order of tasks can be switched without changing the outcome. For example, it does not matter whether you put on your left shoe or right shoe first before heading out to work. As long as you are wearing both shoes when you leave your house.

In mathematics, we say that this situation is commutative- the outcome will be the same no matter the order in which the tasks are done.

Likewise, the **Commutative Property of Addition** states that when two numbers are being added, their order can be changed without affecting the sum. For example, \(30 + 21\) has the same sum as \(21 + 30\).

\[
30 + 21 = 51 \\
21 + 30 = 51
\]

Multiplication behaves in a similar way. The **Commutative Property of Multiplication** states that when two numbers are being multiplied, their order can be changed without affecting the product. For example, \(7 \times 10\) has the same product as \(10 \times 7\).

\[
7 \times 10 = 70 \\
10 \times 7 = 70
\]
Use the commutative law of addition to write the expression $5 + 8 + 5$ in a different way and then find the sum.

Use the commutative law of multiplication to write $2 \cdot 34$ in a different way. Simplify both expressions to show they have identical results.

Using the commutative properties of addition and multiplication, you can reorder a problem so that compatible numbers are next to each other. For example, let us look at the expression $1 + 13 + 9$. The numbers 1 and 9 are compatible because $1 + 9 = 10$ and it is easier to add 10 to a number. We could reorder this problem as

$$1 + 9 + 13 = 10 + 13 = 23$$

Originally, it would have been

$$1 + 13 + 9 = 14 + 9 = 23$$

**YOU TRY:**

Use the commutative properties of addition and multiplication to write the following in different ways.

- **c)** $7 + 9 + 3 + 1$
- **d)** $2 \times 3 \times 5$
It is important to note that subtraction is not commutative. For example, \(4 - 7\) does not have the same difference as \(7 - 4\). However, \(4 - 7\) can be rewritten as \(4 + (-7)\), since subtracting a number is the same as adding its opposite. Once it is rewritten as addition we could use the commutative property and rewrite it as \((-7) + 4\).

Division is not commutative.

**C. Associative Property of Addition and Multiplication**

The **Associative Property of Addition** states that numbers in an addition expression can be grouped in different ways without changing the sum. Below are two ways of simplifying the same addition problem. We have used parentheses to change the grouping.

Original Expression: \(4 + 5 + 6\)

First Grouping: \((4 + 5) + 6 = 9 + 6 = 15\)

Second Grouping: \(4 + (5 + 6) = 4 + 11 = 15\)

In both cases, the sum is the same. This illustrates that changing the grouping of numbers when adding yields the same sum.

**Media Lesson**

**Associative Law of Addition** (Duration 2:09)

*View the video lesson, take notes and complete the problems below.*

Use the associative law of addition to write the expression \((77 + 2) + 3\) in a different way. Simplify both expressions to show they have identical results.

Multiplication has an associative property that works exactly the same. The **Associative Property of Multiplication** states that numbers in a multiplication expression can be regrouped using parentheses. The example below can be rewritten in two different ways using the associative property.

Original Expression: \(-5 \cdot 6 \cdot 2\)

First Grouping: \((-5 \cdot 6) \cdot 2 = (-30) \cdot 2 = -60\)

Second Grouping: \(-5 \cdot (6 \cdot 2) = -5 \cdot (12) = -60\)

The parentheses do not affect the product, the product is the same regardless of where the parentheses are.
Use the associative law of multiplication to write \((12 \cdot 3) \cdot 10\) in a different way. Simplify both expressions to show they have identical results.

**YOU TRY:**

**e)** Rewrite \(7 + 2 + 13\) in two different ways using the associative property. Show that the expressions yield the same answer.

**f)** Rewrite \(\frac{1}{2} \cdot \left(\frac{5}{6} \cdot 6\right)\) using only the associative property.

**D. Inverse Properties of Addition and Multiplication**

What number added to 2 gives 0?

\[
2 + \_ = 0
\]

We know \(2 + (-2) = 0\).

What number added to -6 gives 0?

\[
-6 + \_ = 0
\]

We know \(-6 + (6) = 0\).

In each case the missing number was the opposite of the number. The opposite of a number is its **Additive Inverse**. The **Inverse Property of Addition** states that adding a number and its additive inverse gives zero.

**5 + \_ = 0**
\[ -3 + \_\_ = 0 \]

\[ 1,725,314 + \_\_\_\_\_ = 0 \]

**YOU TRY:**

\( g) \) What is the additive inverse of 13?  
\( h) \) What is the additive inverse of 0.6?

What number multiplied by \( \frac{2}{3} \) gives 1?

\[ \frac{2}{3} \cdot \_\_ = 1 \]

We know \( \frac{2}{3} \cdot \frac{3}{2} = 1 \).

What number multiplied by 2 gives 1?

\[ 2 \cdot \_\_ = 1 \]

We know \( 2 \cdot \frac{1}{2} = 1 \).

In each case, the missing number is the reciprocal of the number. We call the reciprocal of a number its **Multiplicative Inverse**. The **Inverse Property of Multiplication** states that multiplying a number and its multiplicative inverse gives one.
View the video lesson, take notes and complete the problems below.

5 \cdot ____ = 1

217 \cdot ____ = 1

8,345,271 \cdot ________ = 1

YOU TRY:

Find the multiplicative inverse.

i) 9  

j) \(-\frac{1}{9}\)
Exercises

In the following exercises, identify whether each example is using the identity property of addition or multiplication.

1) \[ 101 + 0 = 101 \]  
2) \[ \frac{3}{5} \cdot 1 = \frac{3}{5} \]  
3) \[ -9 \cdot 1 = -9 \]  
4) \[ 0 + 64 = 64 \]  

In the following exercises, find the additive inverse and multiplicative inverse.

5) \[ 8 \]  
6) \[ -17 \]  
7) \[ \frac{7}{12} \]  
8) \[ -\frac{3}{10} \]  

In the following exercises, use the commutative properties to rewrite the given problem.

9) \[ 8 + 9 = \]  
10) \[ 7 + 6 = \]  
11) \[ 7(-13) = \]  
12) \[ (-19)(-14) = \]  
13) \[ -11 + 8 = \]  
14) \[ -15 + 7 = \]  

Identify what property is being used in each problem: identity, commutative, associative, or inverse.

15) \[ 3 \cdot (6 \cdot 3) = (3 \cdot 6) \cdot 3 \]  
16) \[ 55 + 0 = 55 \]  
17) \[ 5 + 9 + 1 = 5 + (9 + 1) \]  
18) \[ 11 \cdot \frac{1}{11} = 1 \]  
19) \[ 17 + (-17) = 0 \]  
20) \[ 2 \cdot 7 \cdot 5 = 2 \cdot 5 \cdot 7 \]
Check your work with the answer key!

Directions: It is very useful to save your math exercise work and use it as a chapter test review when you study for your chapter test and final.

1) Write each question on the screen down to for your record

2) Solve the problem step by step below each question

3) Double check your work to see whether your answer make sense

4) Enter your answer in the answer box in Canvas. Make sure you click on the “Preview” button to make sure you enter the right format before you submit your answer. If you are not sure how to enter your answer with the correct format, ask your instructor.

5) If you did not answer the question correctly, solve the question again from the beginning below your 1st attempt. Sometimes, it is better to start a problem again from the beginning and compare your steps with your 1st attempt to figure out your mistake.

6) Insert your work at the end of each section in your workbook so that you can use it to study for your chapter test later.
SECTION 9.2 Introduction to Variables and Expressions

A. Variables and Algebraic Expressions

An important part of algebra is the variable. A **variable** is a symbol, usually an English letter. It is used to replace an unknown or a changing quantity. Any letter can be used but \( x \) and \( y \) are common.

A **variable** is a symbol, usually an English letter. It is used to replace an unknown or a changing quantity. Any letter can be used but \( x \) and \( y \) are common.

<table>
<thead>
<tr>
<th>Media Lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td>**What is a variable?</td>
</tr>
<tr>
<td>(Duration 3:17)</td>
</tr>
</tbody>
</table>

View the video lesson, follow along and take notes below.

An **algebraic expression** is a mathematical statement that can contain numbers, variables, and operations (addition, subtraction, multiplication, division, etc...). Examples of an algebraic expression would be:

\[
\begin{align*}
x^2 + 4 \\
12 - x \\
x \div 2
\end{align*}
\]

The letter \( x \) in these expressions is a variable.

Note the difference between an expression and equation. An **equation** is a mathematical statement that contains an equal sign. An expression has no equal sign.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 2y - 6 )</td>
<td>( 2y - 6 = 4 )</td>
</tr>
<tr>
<td>( 4 - x )</td>
<td>( 4 - x = -10 )</td>
</tr>
</tbody>
</table>

As we move through this chapter you might notice that the often write variables next to numbers. For example, \( 2y \). This notation represent multiplication. It means “2 times \( y \).” We read it as “two \( y \).”
What options are there to represent multiplication?

________________ ___________________ ___________________

**B. Algebraic Expressions**

We defined an algebraic expression as a mathematical statement that can contain numbers, variables, and operations.

A number in an expression is either a constant or a coefficient. A **constant** is a number that is alone. A **coefficient** accompanies a variable. Consider the following expression:

\[ 3x - 7 \]

In the expression, -7 is a constant because it is alone while 3 is the coefficient of \( x \).

If a variable has no coefficient written in front of it, we assume the coefficient is 1. For example, the expression \( x + 5 \) can be thought of as \( 1x + 5 \).

To **evaluate** an algebraic expression means to find the value of the expression when the variable is replaced by a given number. To evaluate an expression, we **substitute** the given number for the variable in the expression and then simplify the expression using the order of operations.

Example: Evaluate the expression \( 4a + 12 \) when \( a = 4 \).

\[
4a + 12 = 4 \times 4 + 12 \quad \text{Substitue 4 for } a.
\]
\[
= 16 + 12 \quad \text{Follow order of operations to solve.}
\]
\[
= 28
\]
View the video lesson, take notes and complete the problems below.

Variable – A ______________ that represents a ______________________________.

Once we ______________ the value we can ______________ the variable!

Evaluate $3x - 4y^2$ when $x = 2$ and $y = -5$.

Evaluate $\frac{c^2}{3a+4b}$ when $a = 2, b = -3$, and $c = 6$.

View the video lesson, take notes and complete the problems below.

Evaluate the expressions given $x = 4$.

\[
\begin{array}{ccc}
  x + 5 & 2x + 9 & 3x^2 - 17 \\
\end{array}
\]
Evaluate the expressions given $x = 3$ and $y = -1$.

| $x + y$ | $2x - y$ | $\frac{3x - y^2}{2xy}$ |

**YOU TRY:**

Evaluate the expressions given $x = 4$ and $y = -1$.

| a) $2x + y$ | b) $y^2 + 14$ | c) $x - 7 + y$ |

Evaluate the expressions given $a = 5$, $b = -1$, and $c = 2$.

| d) $b^2 - 4ac$ | e) $2a - 5b + 7c$ |
C. Like Terms and Combining Like Terms
An algebraic expression is made up of terms. Terms are constants or the products of numbers and variables. They are separated by addition and subtraction in the expressions. Examples of terms are 7, y, 5x^2, 9a, and 13xy.

\[3x \quad \text{term} \quad - \quad 5y \quad \text{term} \quad + \quad 7 \quad \text{term}\]

In the expression above 3x, 5y, and 7 are terms.

Two terms are like terms if the base variable(s) and exponent on each variable are identical. Below are some examples of like terms.

- 3y and 7y
- 7x^2 and x^2
- 8xy and \(\frac{1}{2}xy\)

Like terms can be combined in an expression. To combine like terms we add or subtract their coefficients and keep the same variable. When you are asked to simplify an expression you are asked to combine like terms.

Example: Simplify \(3x + 2x - 11\).

The common terms are 3x and 2x. These terms are separated by addition, so to combine them we add their coefficients.

\[3x + 2x - 11 \quad \text{We add 3 and 2.} \quad 5x - 11\]

Media Lesson
Identify Like Terms and Combine Like (Duration 4:36)

View the video lesson, take notes and complete the problems below.

Which of these terms are like terms?

\[-2x^3, -2x, 2y, 7x^3, 4y, 6x^2, y^2\]

Simplify each polynomial, if possible.

\[4x^3 - 7x^3 \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 2y^2 + 4y - y^2 + 2 - 9y - 5 + 2y\]
View the video lesson, take notes and complete the problems below.

Example 1: Identify the like terms in each of the following expressions.

\[ 3a - 6a + 10a - a \]
\[ 5x - 10y + 6z - 3x \]
\[ 7n + 3n^2 - 2n^3 + 8n^2 + n - n^3 \]

Example 2: Combine the like terms.

\[ 3a - 6a + 10a - a \]
\[ 5x - 10y + 6z - 3x \]
\[ 7n + 3n^2 - 2n^3 + 8n^2 + n - n^3 \]

YOU TRY:

Simplify the following expression.

f) \[ 4x - x + 7y + 5y - 10 \]

\[ 11 - a - 6a \]
D. Using the Commutative Property
A helpful tool to simplify expressions is to use the commutative property of addition. Recall that the commutative property of addition states that the order we add two numbers is not important. So, $3 + 2$ is the same as $2 + 3$.

When an expression contains more terms, we can rearrange the terms using the commutative property of addition. We could rearrange the following expression before combining like terms.

$$3x + 4y - 2x + 6y$$
$$3x - 2x + 4y + 6y$$

**YOU TRY:**

Simplify the expression by rearranging the terms.

h) $3x + 7 + 4x + 5$

You can decide if you would like to rearrange your terms using the commutative property.
### D. Writing Expressions

In many situations we need to know how to translate word phrases into algebraic expressions. Below is a table of common English words converted into a mathematical expression. You can use this table to assist in translating expressions.

<table>
<thead>
<tr>
<th>Operation</th>
<th>Words</th>
<th>Example</th>
<th>Translation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Addition</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Added to</td>
<td>4 added to $n$</td>
<td>$n + 4$</td>
</tr>
<tr>
<td></td>
<td>More than</td>
<td>2 more than $y$</td>
<td>$y + 2$</td>
</tr>
<tr>
<td></td>
<td>The sum of</td>
<td>The sum of $r$ and $s$</td>
<td>$r + s$</td>
</tr>
<tr>
<td></td>
<td>Increased by</td>
<td>$m$ increased by 6</td>
<td>$m + 6$</td>
</tr>
<tr>
<td></td>
<td>The total of</td>
<td>The total of 8 and $x$</td>
<td>$8 + x$</td>
</tr>
<tr>
<td></td>
<td>Plus</td>
<td>$c$ plus 2</td>
<td>$c + 2$</td>
</tr>
<tr>
<td><strong>Subtraction</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Minus</td>
<td>$x$ minus 1</td>
<td>$x - 1$</td>
</tr>
<tr>
<td></td>
<td>Less than</td>
<td>5 less than $y$</td>
<td>$y - 5$</td>
</tr>
<tr>
<td></td>
<td>Less</td>
<td>4 less $r$</td>
<td>$4 - r$</td>
</tr>
<tr>
<td></td>
<td>Subtracted from</td>
<td>3 subtracted from $t$</td>
<td>$t - 3$</td>
</tr>
<tr>
<td></td>
<td>Decreased by</td>
<td>$m$ decreased by 10</td>
<td>$m - 10$</td>
</tr>
<tr>
<td></td>
<td>The difference</td>
<td>The difference</td>
<td>$x - y$</td>
</tr>
<tr>
<td></td>
<td>between</td>
<td>between $x$ and $y$</td>
<td></td>
</tr>
<tr>
<td><strong>Multiplication</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Times</td>
<td>12 times $x$</td>
<td>$12 \cdot x$</td>
</tr>
<tr>
<td></td>
<td>Of</td>
<td>One-third of $v$</td>
<td>$\frac{1}{3}v$</td>
</tr>
<tr>
<td></td>
<td>The product of</td>
<td>The product of $n$ and $k$</td>
<td>$nk$ or $n \cdot k$</td>
</tr>
<tr>
<td></td>
<td>Multiplied by</td>
<td>$y$ multiplied by 3</td>
<td>$3y$</td>
</tr>
<tr>
<td></td>
<td>Twice</td>
<td>Twice $d$</td>
<td>$2d$ or $2 \cdot d$</td>
</tr>
<tr>
<td><strong>Division</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Divided by</td>
<td>$n$ divided by 4</td>
<td>$\frac{n}{4}$</td>
</tr>
<tr>
<td></td>
<td>The quotient of</td>
<td>The quotient of $t$ and $x$</td>
<td>$\frac{t}{x}$</td>
</tr>
</tbody>
</table>
Let us first review.

**Media Lesson**

Translating Word Statement to Math (Duration 3:54)

View the video lesson, take notes and complete the problems below.

The sum of 3 and 8.  

The sum of 7 and 12.  

The difference of 7 and 12.

The difference of \(x\) and \(y\).  

The difference of \(n\) and 4.

The sum of five and three is eight.

The difference of two and seven is negative five.

The sum of five, three, and six is fourteen.

The difference of nine, three, and four is two.

**Media Lesson**

Ex: Write an Algebraic Expression in the Form \(x+c\) and \(c-x\) (Duration 1:12)

View the video lesson, take notes and complete the problems below.

Write this English phrase as an algebraic expression. Let the variable \(x\) represent the number.

*seven more than a number*

Write this English phrase as an algebraic expression. Let the variable \(x\) represent the number.

*4 less than a number*
<table>
<thead>
<tr>
<th>YOU TRY: Write the phrase as an algebraic example.</th>
</tr>
</thead>
<tbody>
<tr>
<td>i) The difference of 17 and a number</td>
</tr>
<tr>
<td>j) The sum of a number, 3, and 6</td>
</tr>
</tbody>
</table>
**EXERCISES**

In the following exercises, evaluate the expression for the given value.

<p>| | | | | | | | | |</p>
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</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$7x + 8$ when $x = 2$</td>
<td>2</td>
<td>$9x + 7$ when $x = 3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$8x - 6$ when $x = 7$</td>
<td>4</td>
<td>$x^2$ when $x = 12$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$x^2 + 3x - 7$ when $x = 4$</td>
<td>6</td>
<td>$2x - 4y - 5$ when $x = 6$ and $y = 9$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>$\frac{a}{b+1}$ when $a = 10$ and $b = 4$</td>
<td>8</td>
<td>$\frac{3a}{2b+1}$ when $a = 11$ and $b = 1$</td>
<td></td>
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</tr>
</tbody>
</table>

In the following exercises, identify all sets of like terms.

<p>| | | | | | | | | |</p>
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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>$-3x$, $5$, $x^2$, $11$, $18x$</td>
<td>10</td>
<td>$2y$, $y^2$, $x$, $11$, $5y$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>$2x$, $15y$, $-9x$, $y$, $2y$</td>
<td>12</td>
<td>$8a$, $5a^2$, $3$, $17a$, $a^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In the following exercises, simplify the given expression by combining like terms.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>13</strong></td>
<td>$10x + 3x$</td>
<td><strong>14</strong></td>
</tr>
<tr>
<td><strong>15</strong></td>
<td>$2x + 7 - 6x$</td>
<td><strong>16</strong></td>
</tr>
<tr>
<td><strong>17</strong></td>
<td>$9x + 3x + 8$</td>
<td><strong>18</strong></td>
</tr>
<tr>
<td><strong>19</strong></td>
<td>$8d + 6 + 2d + 5$</td>
<td><strong>20</strong></td>
</tr>
<tr>
<td><strong>21</strong></td>
<td>$10a + 7 + 5a - 2 + 7a - 4$</td>
<td><strong>22</strong></td>
</tr>
</tbody>
</table>
Check your work with the answer key!

Online Quiz

Log on to Canvas to take the section quiz

Directions: It is very useful to save your math exercise work and use it as a chapter test review when you study for your chapter test and final.

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8) Solve the problem step by step below each question

9) Double check your work to see whether your answer make sense

10) Enter your answer in the answer box in Canvas. Make sure you click on the “Preview” button to make sure you enter the right format before you submit your answer. If you are not sure how to enter your answer with the correct format, ask your instructor.

11) If you did not answer the question correctly, solve the question again from the beginning below your 1st attempt. Sometimes, it is better to start a problem again from the beginning and compare your steps with your 1st attempt to figure out your mistake.

12) Insert your work at the end of each section in your workbook so that you can use it to study for your chapter test later.
Section 9.3 Solving Equations

A. Solving One Step Equations

An equation is a mathematical statement that uses an equal sign. The following are equations:

\[ x + 1 = 10 \]
\[ 3y - 10 = 2 \]
\[ \frac{x}{4} = 11 \]

When we solve an equation, we are looking for the value of our variable that would make that statement true. We call this value the solution of the equation.

Can you recognize the solution to \( x + 1 = 10 \)? The solution is 9. We say 9 is a solution to \( x + 1 = 10 \) because when we substitute 9 for \( x \) the resulting statement is true.

\[
\begin{align*}
x + 1 & = 10 \\
9 + 1 & = 10 \\
10 & = 10 \checkmark
\end{align*}
\]

Sometimes the answer is not as obvious. We solve an equation by a process called isolating the variable. When we isolate a variable, we “unattach” any constants and coefficients that are on the same side of the equal sign as our variable. We make use of opposite operations—addition and subtraction are opposites and multiplication and division are opposites.

**Addition and Subtraction**

Example: Solve \( y - 10 = 2 \).

We begin by unattaching the constant 10. In the equation our constant is being subtracted, to unattach it we do the opposite operation, addition. We add 10 to both sides of our equation.

\[
\begin{align*}
y - 10 & = 2 \\
+10 & \quad +10 \\
y & = 12
\end{align*}
\]

Our solution is \( y = 12 \). You can check this is correct because \( 12 - 10 = 2 \).
Media Lesson
Ex: Solve One Step Equations by Add and Subtract Whole Numbers
(Duration 5:00)

View the video lesson, take notes and complete the problems below.

Examples: Solve.

\[ x + 9 = 24 \quad x - 7 = 32 \]

Media Lesson
Ex: Solving One Step Equation by Add/Subtracting Integers
(Duration 3:17)

View the video lesson, take notes and complete the problems below.

Examples: Solve.

\[ x + 7 = -24 \quad x - 9 = -21 \]

YOU TRY:

Solve. Substitute your answer back into the equation to check your solution.

k) \[ x + 4 = 15 \]

l) \[ y - 7 = 15 \]
**Multiplication and Division**

Example: Solve $3y = 12$.

We want to isolate the variable $y$. We begin by unattaching the coefficient that accompanies it. In the equation $y$ is being multiplied by the coefficient $3$. The opposite of multiplication is division. We divide by $3$ on both sides of the equation.

\[
\frac{3y}{3} = \frac{12}{3}
\]

\[
y = 4
\]

Our solution is $y = 4$. You can check this is correct because $3 \times 4 = 12$.

---

**Media Lesson**

Ex: Solve One Step Equation By Mult And Div Whole Numbers (Duration 2:53)

View the video lesson, take notes and complete the problems below.

Examples: Solve.

\[
5x = 35
\]

\[
\frac{x}{3} = 36
\]

---

**Media Lesson**

Ex: Solving One Step Equation by Mult/Div Integers (Duration 2:45)

View the video lesson, take notes and complete the problems below.

Examples: Solve.

\[
-6x = 36
\]

\[
-\frac{x}{3} = -8
\]
Often we might get close to solving for $x$ only to get a solution like this $-x = 3$, where a negative sign is in front of our variable. To get our final answer we can multiply both sides by -1. This gives us

$$ -x = 3 $$

$$ -1(-x) = -1(3) $$

$$ x = -3 $$

A shortcut is to just change all signs to their opposites. For example, for $-x = -4$ the $x$ and 4 are negative. We change them both to positive and get $x = 4$.

**YOU TRY:**

Solve. Substitute your answer back into the equation to check your solution.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>m) 5$x$ = 25</td>
<td>n) $-3a = -36$</td>
</tr>
</tbody>
</table>

**Fractions**

Solving an equation with fractions will have the same steps as solving our previous equations. The only difference is when we have a fraction as the coefficient of our variable, we can use the reciprocal to unattach the coefficient.

Example: Solve $\frac{1}{2}x = 13$

The coefficient of $x$ is $\frac{1}{2}$. We will multiply by the reciprocal $\frac{1}{\frac{1}{2}}$ on both sides of the equation.

$$ \frac{1}{2}x = 13 $$

$$ \frac{2}{1} \left( \frac{1}{2}x \right) = (13) \frac{2}{1} \quad \text{Multiply by the reciprocal} $$

$$ x = (13)2 $$

$$ x = 26 $$
Media Lesson
Solve One Step Equations with Fractions (Duration 8:33)

View the video lesson, take notes and complete the problems below.

\[
\begin{align*}
  x + \frac{1}{20} &= \frac{1}{5} \\
  x - \frac{2}{3} &= \frac{3}{4}
\end{align*}
\]

\[
\begin{align*}
  \frac{3}{4}x &= \frac{15}{28} \\
  6x &= \frac{4}{15}
\end{align*}
\]

\[
\begin{align*}
  -\frac{4}{9}x &= \frac{14}{15}
\end{align*}
\]

YOU TRY:

Solve.

k) \( \frac{3}{4}a = 8 \)

l) \( n - \frac{1}{4} = 8 \frac{1}{2} \)
Combining Like Terms
It makes it easier to solve an equation if we combine like terms. If there are like terms on the same side of the equal sign, we begin by combining them before isolating the variable.

View the video lesson, take notes and complete the problems below.
Always __________________ each side before ___________________.

5x + 3 − 4x = 7       15 − 7 = 8x − 6x

YOU TRY:
Solve. Substitute your answer back into the equation to check your solution.

\[ p) \ x + 8 = 12 − 5 \]
\[ q) \ 7x − 3x = 70 − 46 \]
B. Solving Two-Step Equations

When solving equations it helps to think about how each thing is attached and how order of operations would be applied. We need to “undo” the order of operations, so we work backwards.

We first undo addition and subtraction. Then we undo multiplication and division.

Simplifying we use order of operations and we ________________ before we ________________.

Solving we work in reverse so we will ________________ first and then ________________ second.

\[5x - 7 = 8\] \[\quad \quad \quad -9 = -5 - 2x\]

A fraction bar is the same as ________________.

To clear division we will ________________ both sides by the ________________.

Because we are solving and working backwards to our solution we will ________________ and then ________________.

\[\frac{x}{4} - 3 = 7\] \[\quad \quad \quad -2 = 4 + \frac{x}{6}\]
### YOU TRY:

Solve. Substitute your answer back into the equation to check your solution.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>r)</strong> $5x + 7 = 37$</td>
<td><strong>s)</strong> $-7 + 2x = -9$</td>
</tr>
<tr>
<td><strong>t)</strong> $\frac{x}{4} + 1 = -7$</td>
<td><strong>u)</strong> $10 + \frac{x}{3} = 21$</td>
</tr>
</tbody>
</table>
C. Solving Multi-Step Equations

If there is a variable on both sides of the equal sign, we must first get our variables on the same side of the equation. We move the variable with the smaller coefficient. To move it we add its additive inverse to both sides.

Example: Solve $2x + 7 = -3x + 22$

There is an $x$ on both sides of the equal sign. The first step is to move $-3x$ since it has the smaller coefficient. The additive inverse of $-3x$ is $+3x$.

\[
\begin{align*}
6x + 4 &= -3x + 22 \\
+3x +3x &= Move \ variables \ to \ same \ side. \\
9x + 4 &= 22 \\
-4 -4 &= Undo \ addition. \\
9x &= 18 \\
\frac{9}{9} &= Undo \ multiplication. \\
x &= 2
\end{align*}
\]

The guidelines to solve algebraic equations are:

1) Collect like terms that are on the same side of the equal sign.
2) Move variables to the same side of the equal sign.
3) Undo addition and subtraction.
4) Undo multiplication and division.

Media Lesson

Solve an Equation with Variables on Both Sides (Duration 3:55)

View the video lesson, take notes and complete the problems below.

Solve $7x - 2 = 3x + 18$
YOU TRY:

Solve.

v) \[ 2x + 7 = -3x + 22 \]

w) \[ 6n - 2 = -3n + 7 \]

D. Writing Equations

We can combine our knowledge of solving an equation and writing variable expression to solve word problems. An equation has an equal sign. So if we have a sentence that tells us that two phrases are equal, we can translate it into an equation. We look for clue words that mean equals.

<table>
<thead>
<tr>
<th>Equals</th>
<th>Is</th>
<th>Are</th>
<th>Equal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gives</td>
<td>Is equal to</td>
<td>Is equivalent to</td>
</tr>
<tr>
<td></td>
<td>Yields</td>
<td>Results in</td>
<td>was</td>
</tr>
</tbody>
</table>

Example: Write an equation for the following and then solve.

Twice a number is 16, find the number.

We first will translate it to an equation by finding key terms.

\[
\frac{\text{Twice a number is}}{2 \text{ times}} \quad \frac{16, \text{find the number}}{x} =
\]

The location of “is” tells us the equal sign is between \( x \) and 16.

\[ 2x = 16 \]

We have our equation. We can now solve.

\[
\frac{2x}{2} = \frac{16}{2} \\
x = 8
\]

This answer makes sense because twice of 8 is 16.
YOU TRY:

Translate the sentence into an algebraic equation.

x) Three more than $x$ is equal to 47.
## Exercises

Solve the following equations.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1)</strong> $x + 7 = 17$</td>
<td><strong>2)</strong> $x + 8 = 22$</td>
</tr>
<tr>
<td><strong>3)</strong> $x - 13 = 10$</td>
<td><strong>4)</strong> $x - 4 = 15$</td>
</tr>
<tr>
<td><strong>5)</strong> $x + 21 = -15$</td>
<td><strong>6)</strong> $x - 15 = -42$</td>
</tr>
<tr>
<td><strong>7)</strong> $x + \frac{2}{5} = \frac{4}{5}$</td>
<td><strong>8)</strong> $x - \frac{1}{3} = 2$</td>
</tr>
</tbody>
</table>

Solve the following equations.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>9)</strong> $2x = 12$</td>
<td><strong>10)</strong> $3x = 24$</td>
</tr>
<tr>
<td><strong>11)</strong> $-4x = 16$</td>
<td><strong>12)</strong> $-7x = 42$</td>
</tr>
<tr>
<td><strong>13)</strong> $-72 = -12y$</td>
<td><strong>14)</strong> $8x = -56$</td>
</tr>
<tr>
<td><strong>15)</strong> $\frac{x}{4} = 15$</td>
<td><strong>16)</strong> $\frac{x}{2} = 14$</td>
</tr>
<tr>
<td>Equation</td>
<td>Solution</td>
</tr>
<tr>
<td>-------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>( \frac{x}{-3} = -12 )</td>
<td></td>
</tr>
<tr>
<td>( \frac{x}{9} = -6 )</td>
<td></td>
</tr>
<tr>
<td>( \frac{3}{5}x = 15 )</td>
<td></td>
</tr>
<tr>
<td>( -\frac{5}{8}x = 40 )</td>
<td></td>
</tr>
</tbody>
</table>

Solve the following equations by combining like terms.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 5x - 3x = 10 )</td>
<td></td>
</tr>
<tr>
<td>( -4x - 7x = -33 )</td>
<td></td>
</tr>
<tr>
<td>( 5x = -72 + 47 )</td>
<td></td>
</tr>
<tr>
<td>( 9x = -42 - 3 )</td>
<td></td>
</tr>
</tbody>
</table>
Solve the following two step equations. Check your answers.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>25</strong></td>
<td>22 = 5x + 7</td>
</tr>
<tr>
<td><strong>26</strong></td>
<td>7x − 2 = −51</td>
</tr>
<tr>
<td><strong>27</strong></td>
<td>−50 = 8x − 10</td>
</tr>
<tr>
<td><strong>28</strong></td>
<td>−41 = 9x − 5</td>
</tr>
<tr>
<td><strong>29</strong></td>
<td>4 − 3x = 22</td>
</tr>
<tr>
<td><strong>30</strong></td>
<td>3 − 8x = −53</td>
</tr>
<tr>
<td><strong>31</strong></td>
<td>5x − 3x + 2 = 18</td>
</tr>
<tr>
<td><strong>32</strong></td>
<td>−7x − 2x + 7 = −11</td>
</tr>
</tbody>
</table>

Solve the following equations. Check your answers.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>33</strong></td>
<td>6x − 15 = 5x + 3</td>
</tr>
<tr>
<td><strong>34</strong></td>
<td>4x − 17 = 3x + 2</td>
</tr>
<tr>
<td><strong>35</strong></td>
<td>4x + 5 = −x − 40</td>
</tr>
<tr>
<td><strong>36</strong></td>
<td>9x + 7 = −2x − 37</td>
</tr>
<tr>
<td><strong>37</strong></td>
<td>5x − 6 = −2x + 15</td>
</tr>
<tr>
<td><strong>38</strong></td>
<td>4x − 3 = 8x + 9</td>
</tr>
<tr>
<td><strong>39</strong></td>
<td>7x − 3x + 2 = 5 − 2x + 9</td>
</tr>
<tr>
<td><strong>40</strong></td>
<td>−2x + 7 − 12 + 3x = 5x − 7x + 10</td>
</tr>
</tbody>
</table>
In the following exercises, translate to an equation and then solve.

<table>
<thead>
<tr>
<th>41) Five more than $x$ is equal to 21.</th>
<th>42) The sum of $x$ and -5 is 33.</th>
</tr>
</thead>
<tbody>
<tr>
<td>43) Three less than $y$ is $-19$.</td>
<td>44) The sum of $y$ and $-3$ is 40.</td>
</tr>
<tr>
<td>45) The difference of $9x$ and $8x$ is 17.</td>
<td>46) The difference of $5c$ and $4c$ is 60.</td>
</tr>
</tbody>
</table>
Check your work with the answer key!

Online Quiz

Log on to Canvas to take the section quiz

Directions: It is very useful to save your math exercise work and use it as a chapter test review when you study for your chapter test and final.

1) Write each question on the screen down to for your record

2) Solve the problem step by step below each question

3) Double check your work to see whether your answer make sense

4) Enter your answer in the answer box in Canvas. Make sure you click on the “Preview” button to make sure you enter the right format before you submit your answer. If you are not sure how to enter your answer with the correct format, ask your instructor.

5) If you did not answer the question correctly, solve the question again from the beginning below your 1st attempt. Sometimes, it is better to start a problem again from the beginning and compare your steps with your 1st attempt to figure out your mistake.

6) Insert your work at the end of each section in your workbook so that you can use it to study for your chapter test later.
### CHAPTER REVIEW

#### KEY TERMS AND CONCEPTS

Look for the following terms and concepts as you work through the workbook. In the space below, explain the meaning of each of these concepts and terms in your own words. Provide examples that are not identical to those in the text or in the media lesson.

<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identity Property of Addition</td>
<td></td>
</tr>
<tr>
<td>Identity Property of Multiplication</td>
<td></td>
</tr>
<tr>
<td>Commutative Property of Addition</td>
<td></td>
</tr>
<tr>
<td>Commutative Property of Multiplication</td>
<td></td>
</tr>
<tr>
<td>Associative Property of Addition</td>
<td></td>
</tr>
<tr>
<td>Associative Property of Multiplication</td>
<td></td>
</tr>
<tr>
<td>Additive Inverse</td>
<td></td>
</tr>
<tr>
<td>Inverse Property of Addition</td>
<td></td>
</tr>
<tr>
<td>Multiplicative Inverse</td>
<td></td>
</tr>
<tr>
<td>Inverse Property of Multiplication</td>
<td></td>
</tr>
<tr>
<td>Variable</td>
<td></td>
</tr>
<tr>
<td>Algebraic Expression</td>
<td></td>
</tr>
<tr>
<td>Equation</td>
<td></td>
</tr>
<tr>
<td>Coefficient</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>Evaluate</td>
</tr>
<tr>
<td>----------</td>
<td>----------</td>
</tr>
</tbody>
</table>

Identify what property of algebra is being used.

1) \(-5 + 5 = 0\)  
2) \(2 + 56 = 56 + 2\)

3) \(\frac{1}{2} \cdot (2 \cdot 5) = \left(\frac{1}{2} \cdot 2\right) \cdot 5\)  
4) \(1,345 + 0 = 1,345\)

5) \(\frac{1}{2} \cdot 2 = 1\)  
6) \(3 \cdot 75 = 75 \cdot 3\)

In the following exercises, evaluate the expression for the given values.

7) \(9x - 2\) when \(x = 5\)  
8) \(8x - 3\) when \(x = 2\)

9) \(2^x\) when \(x = 2\)  
10) \(2x + 5y - 4\) when \(x = 11\) and \(y = 3\)

11) \(5x - 2y - 9\) when \(x = 7\) and \(y = 8\)  
12) \(5x - 4\) when \(x = 6\)

In the following exercises, simplify the expression.

13) \(17a + 9a\)  
14) \(18z - 9z\)

15) \(7u + 2 + 3u + 1\)  
16) \(6y - 4y + y\)

17) \(8x + 7 + 4x - 5\)  
18) \(x + 10x\)
In the following exercises, solve the equation.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>19)</td>
<td>( y + 5 = -6 )</td>
<td>20)</td>
</tr>
<tr>
<td>21)</td>
<td>( a + 25 = 45 )</td>
<td>22)</td>
</tr>
<tr>
<td>23)</td>
<td>( y - 72 = 5 )</td>
<td>24)</td>
</tr>
<tr>
<td>25)</td>
<td>( c + \frac{2}{6} = \frac{4}{6} )</td>
<td>26)</td>
</tr>
</tbody>
</table>

In the following exercises, solve the equation.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>27)</td>
<td>( c + 3 - 10 = 18 )</td>
<td>28)</td>
</tr>
<tr>
<td>29)</td>
<td>( 8a + 3a - 6a = 10 )</td>
<td>30)</td>
</tr>
<tr>
<td>31)</td>
<td>( 7x - 6x = 10 + 23 )</td>
<td>32)</td>
</tr>
</tbody>
</table>

In the following exercises, solve the equation.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>33)</td>
<td>( 8x = 32 )</td>
<td>34)</td>
</tr>
<tr>
<td>35)</td>
<td>( -9x = -27 )</td>
<td>36)</td>
</tr>
<tr>
<td>37)</td>
<td>( -5c = 55 )</td>
<td>38)</td>
</tr>
<tr>
<td>39)</td>
<td>( \frac{x}{5} = -8 )</td>
<td>40)</td>
</tr>
<tr>
<td>41)</td>
<td>( \frac{2}{3} y = 18 )</td>
<td>42)</td>
</tr>
</tbody>
</table>

In the following exercises, solve the equation.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>43)</td>
<td>( 7x - 8 = 34 )</td>
<td>44)</td>
</tr>
<tr>
<td>45)</td>
<td>( 14y + 7 = 91 )</td>
<td>46)</td>
</tr>
<tr>
<td>47)</td>
<td>( -50 = 7n - 1 )</td>
<td>48)</td>
</tr>
<tr>
<td>49)</td>
<td>( \frac{x}{2} - 10 = 2 )</td>
<td>50)</td>
</tr>
</tbody>
</table>

In the following exercises, solve the equation.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>51)</td>
<td>( 9k = 8k - 11 )</td>
<td>52)</td>
</tr>
<tr>
<td>53)</td>
<td>( 5x = 41 - 6 )</td>
<td>54)</td>
</tr>
<tr>
<td>55)</td>
<td>( 26 + 8d = 9d + 11 )</td>
<td>56)</td>
</tr>
<tr>
<td>57)</td>
<td>( 12x - 17 = -3x + 13 )</td>
<td>58)</td>
</tr>
</tbody>
</table>

In the following exercises, translate to an equation and then solve.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>59)</td>
<td>Five more than ( x ) in equal to 21.</td>
<td>60)</td>
</tr>
<tr>
<td>61)</td>
<td>Ten less than ( m ) is (-14).</td>
<td>62)</td>
</tr>
<tr>
<td>63)</td>
<td>Fifteen more than ( x ) is 21.</td>
<td>64)</td>
</tr>
</tbody>
</table>