CHAPTER 4: LINEAR EQUATION APPLICATIONS

Chapter Objectives

By the end of this chapter, student should be able to

✓ Translate sentences into equation
✓ Model and solve: - Discount and mark-up problems
   - Geometry problem with perimeter and triangles
   - Investment problems
   - Mixture problems
   - Uniform motion problems

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Word problems can be tricky. The goal is becoming proficient in translating an English sentence into a mathematical sentence. In this chapter, we focus on word problems modeled by a linear equation and solve.

Below is a table of common English words converted into a mathematical expression. You can use this table to assist in translating expressions and equations.

<table>
<thead>
<tr>
<th>Operation</th>
<th>Words</th>
<th>Example</th>
<th>Translation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Addition</strong></td>
<td>Added to</td>
<td>4 added to ( n )</td>
<td>( n + 4 )</td>
</tr>
<tr>
<td></td>
<td>More than</td>
<td>2 more than ( y )</td>
<td>( y + 2 )</td>
</tr>
<tr>
<td></td>
<td>The sum of</td>
<td>The sum of ( r ) and ( s )</td>
<td>( r + s )</td>
</tr>
<tr>
<td></td>
<td>Increased by</td>
<td>( m ) increased by 6</td>
<td>( m + 6 )</td>
</tr>
<tr>
<td></td>
<td>The total of</td>
<td>The total of 8 and ( x )</td>
<td>( 8 + x )</td>
</tr>
<tr>
<td></td>
<td>Plus</td>
<td>( c ) plus 2</td>
<td>( c + 2 )</td>
</tr>
<tr>
<td><strong>Subtraction</strong></td>
<td>Minus</td>
<td>( x ) minus 1</td>
<td>( x - 1 )</td>
</tr>
<tr>
<td></td>
<td>Less than</td>
<td>5 less than ( y )</td>
<td>( y - 5 )</td>
</tr>
<tr>
<td></td>
<td>Less</td>
<td>4 less ( r )</td>
<td>( 4 - r )</td>
</tr>
<tr>
<td></td>
<td>Subtracted from</td>
<td>3 subtracted from ( t )</td>
<td>( t - 3 )</td>
</tr>
<tr>
<td></td>
<td>Decreased by</td>
<td>( m ) decreased by 10</td>
<td>( m - 10 )</td>
</tr>
<tr>
<td></td>
<td>The difference between</td>
<td>The difference between ( x ) and ( y )</td>
<td>( x - y )</td>
</tr>
<tr>
<td><strong>Multiplication</strong></td>
<td>Times</td>
<td>12 times ( x )</td>
<td>( 12 \cdot x )</td>
</tr>
<tr>
<td></td>
<td>Of</td>
<td>One-third of ( v )</td>
<td>( \frac{1}{3} v )</td>
</tr>
<tr>
<td></td>
<td>The product of</td>
<td>The product of ( n ) and ( k )</td>
<td>( n k ) or ( n \cdot k )</td>
</tr>
<tr>
<td></td>
<td>Multiplied by</td>
<td>( y ) multiplied by 3</td>
<td>( 3y )</td>
</tr>
<tr>
<td></td>
<td>Twice</td>
<td>Twice ( d )</td>
<td>( 2d ) or ( 2 \cdot d )</td>
</tr>
<tr>
<td><strong>Division</strong></td>
<td>Divided by</td>
<td>( n ) divided by 4</td>
<td>( \frac{n}{4} )</td>
</tr>
<tr>
<td></td>
<td>The quotient of</td>
<td>The quotient of ( t ) and ( x )</td>
<td>( \frac{t}{x} )</td>
</tr>
<tr>
<td></td>
<td>The ratio of</td>
<td>The ratio of ( x ) to ( p )</td>
<td>( \frac{x}{p} )</td>
</tr>
<tr>
<td></td>
<td>Per</td>
<td>2 per ( b )</td>
<td>( \frac{2}{b} )</td>
</tr>
<tr>
<td><strong>Power</strong></td>
<td>The square of</td>
<td>The square of ( y )</td>
<td>( y^2 )</td>
</tr>
<tr>
<td></td>
<td>The cube of</td>
<td>The cube of ( k )</td>
<td>( k^3 )</td>
</tr>
<tr>
<td><strong>Equals</strong></td>
<td>Is</td>
<td>Are</td>
<td>Equal</td>
</tr>
<tr>
<td></td>
<td>Gives</td>
<td>Is equal to</td>
<td>Is equivalent to</td>
</tr>
<tr>
<td></td>
<td>Yields</td>
<td>Results in</td>
<td>was</td>
</tr>
</tbody>
</table>
SECTION 4.1: INTEGER PROBLEMS

A. NUMBER PROBLEMS

View the video lesson, take notes and complete the problems below.

Translate:
- Is/Were/Was/Will Be: ___________
- More than: __________________________________________
- Subtracted from/Less than: _____________________________

Example:

a) Five less than three times a number is nineteen. What is the number?

b) Seven more than twice a number is six less than three times the same number. What is the number?

YOU TRY

a) If 28 less than five times a number is 232, what is the number?

b) Fifteen more than three times a number is the same as ten less than six times the number. What is the number?

B. CONSECUTIVE INTEGERS

Another type of number problem involves consecutive integers.

Definition

Consecutive integers are integers that come one after the other, such as... 3, 4, 5, and so on (or equivalently, −3, −2, −1, ...).

- If we are trying to find several consecutive integers, it is important to identify the first integer and then assign all the following integers. E.g., if \( x \) is the first integer, then \( x + 1 \) will be the next, and \( x + 2 \) will be the following, and so on.

- If we are trying to find several even or odd consecutive integers, it is important to identify the first integer and then assign all the following even or odd integers. E.g., if \( x \) is the first integer, then \( x + 2 \) will be the next, and \( x + 4 \) will be the following, and so on.
View the video lesson, take notes and complete the problems below.

Consecutive Numbers: ________________________________

First: _________________________________________

Second: ________________________________________

Third: _________________________________________

Example:

a) Find three consecutive numbers whose sum is 543.

b) Find four consecutive integers whose sum is −222.

YOU TRY

a) The sum of three consecutive positive integers is 93. What are the positive integers?

b) The sum of three consecutive even positive integers is 246. What are the numbers?

c) Find three consecutive odd positive integers so that the sum of twice the first integer, the second integer, and three times the third integer is 152.
EXERCISES

1) When five is added to three more than a certain number, the result is 19. What is the number?

2) If five is subtracted from three times a certain number, the result is 10. What is the number?

3) When 18 is subtracted from six times a certain number, the result is −42. What is the number?

4) A certain number added twice to itself equals 96. What is the number?

5) A number plus itself, plus twice itself, plus 4 times itself, is equal to −104. What is the number?

6) Sixty more than nine times a number is the same as two less than ten times the number. What is the number?

7) Eleven less than seven times a number is five more than six times the number. Find the number.

8) Fourteen less than eight times a number is three more than four times the number. What is the number?

9) The sum of three consecutive integers is 108. What are the integers?

10) The sum of three consecutive integers is −126. What are the integers?

11) Find three consecutive integers such that the sum of the first, twice the second, and three times the third is −76.

12) The sum of two consecutive even integers is 106. What are the integers?

13) The sum of three consecutive odd integers is 189. What are the integers?

14) The sum of three consecutive odd integers is 255. What are the integers?

15) Find three consecutive odd integers such that the sum of the first, two times the second, and three times the third is 70.

Log on to Canvas to take the section quiz
SECTION 4.2: MARK-UP AND DISCOUNT PROBLEMS

When paying for our meal at a restaurant, we do not pay just the price of the food. We also pay a percentage for sales tax. Imagine our food cost $65, but the sales tax is 8%. Then we would pay the original $65 plus 8% of that $65. In mathematical terms, the final price would be

\[ \text{food cost} + \text{sales tax} = \text{total bill} \]

\[ 65 + (0.08) \times 65 = 70.20 \]

Likewise, if we were buying an item on sale, we would not pay the original price of the item, but a new lower price based on a percentage of the original. Suppose we want to buy a $38 sweater, and it’s on sale for 15% off. We would take 15% of the $38 off from the original $38. In mathematical terms, the final price would be

\[ \text{cost of sweater} - \text{discount} = \text{final price} \]

\[ 38 - (0.15) \times 38 = 32.30 \]

Some of us may not be familiar with retail, but a business must first acquire merchandise before the business sells it to the public. They add, to their original cost, a percentage of that cost to make a profit when they sell an item. Consider a business owner that purchases a vase for $350. The owner marks up the price by 25% to sell the vase at a higher price and make a profit. In mathematical terms, the final price to the consumer would be

\[ \text{cost to the owner} + \text{mark-up} = \text{final price} \]

\[ 350 + (0.25) \times 350 = 437.50 \]

Notice how these scenarios have the same format:

\[ \text{Original Price} \pm \text{Percent} \cdot \text{Original Price} = \text{Final Price} \]

The plus or minus is determined by whether we are increasing or decreasing the original price. In these previous three scenarios, the discount problem was the only time we were decreasing the original price. We will be dealing with two prices. We first need to determine which price came first in the timeline because that price is the original.

A. MARK-UP PROBLEMS

Mark-up formula

<table>
<thead>
<tr>
<th>Given the original cost of an item ( C ), the mark-up rate ( r ), the selling price of the item including the mark-up is given by</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \text{Selling price} = \text{Original cost} + (\text{Mark-up rate})(\text{Original cost}) ]</td>
</tr>
<tr>
<td>[ S = C + rC ]</td>
</tr>
</tbody>
</table>

where \( S \) is the selling price.

Note, the mark-up rate is a percentage and should be converted to a decimal when using the formula.
View the video lesson, take notes and complete the problems below.

Mark-up Formula

\[ \frac{S - C}{C} \]

S _______________________

C _________________________________________________________________

r _______________________________________________________________________________________

Example:

a) The cost to a distributor for a product is $34. The distributor sells the product for $58 to his customers. What is the mark-up rate? Be sure to round your answer to the nearest whole percent, e.g. 0.435 would be 44%.

YOU TRY

a) A retailer acquired a laptop for $2,015 and sold it for $3,324.75. What was the percent markup?
B. DISCOUNT PROBLEMS

Discount formula

Given the original cost of an item $R$, the discount rate $r$, the sale price of the item is given by

\[ S = R - rR \]

where $S$ is the sale price.

Note, the discount rate is a percentage and should be converted to a decimal when using the formula.

View the video lesson, take notes and complete the problems below.

Discount Formula

\[
\begin{align*}
S & \quad \text{________________________________________________}_
\end{align*}
\]

\[
\begin{align*}
R & \quad \text{________________________________________________}_
\end{align*}
\]

\[
\begin{align*}
\text{r} & \quad \text{________________________________________________}_
\end{align*}
\]

Example:

a) The sale price of a product is $220, which is 40% off the regular price. Find the regular price. Be sure to round your answer to the nearest cent.

YOU TRY

a) Sue bought a sweater for $307.70 after a 15% discount. How much was it before the discount?
EXERCISES

1) A jewelry storeowner bought a diamond in St. Thomas. He then sold it in the United States for $5,500. If he sold it at an 87% mark-up, what was his original cost for the diamond in St. Thomas? Round your answer to the nearest cent.

2) Sari bought a $125 pair of shoes on sale for $93.75. What was the percent discount?

3) Mickey wants to buy her first new car, but she only has $35,500 to spend. She has her eye on a $34,000 hybrid. However, taxes, license and registration tack on an extra 9.5% of the purchase price. What would her actual cost be? Can she afford this car?

4) Rusty took his family out to eat at a nice restaurant. The bill was $282.75 after a sales tax of 8.75%. How much was the food before taxes?

5) As a sales representative for Nerds-Are-Us, Mal gets an 18% employee discount. If she purchased a tablet for $705.20, what was the price before the discount?

6) Jay found a rare book at an estate sale and purchased it for $350. He then sold it online for $770. What was his percent markup?

7) María found a computer for $4,500. If the sales tax is 6.5%, what did she pay for the computer?

8) Bella’s clothing store pays $7.25 per pair of earrings. The store markups 175%. How much is the selling price for a pair of earrings? Round your answer to the nearest cent.

9) The Fancy steakhouse markups their steaks 325%. Each piece of premium steak cost $12.00 at wholesale price. What is the selling price of a piece of steak?

Log on to Canvas to take the section quiz
SECTION 4.3: GEOMETRY PROBLEMS

Another example of translating English sentences to mathematical sentences comes from geometry. We will discuss triangles and perimeter problems.

A. TRIANGLES

**Sum of Angles in a Triangle**

Given a triangle with three angles. The sum of the three angles is \(180^\circ\). I.e., if the angles in a triangle are \(a^\circ\), \(b^\circ\), and \(c^\circ\), then

\[
a^\circ + b^\circ + c^\circ = 180^\circ
\]

The sum of the three interior angles of a triangle of any size is always 180 degrees.

**MEDIA LESSON**

Sum of angles in a triangle (Duration 4:59)

View the video lesson, take notes and complete the problems below.

Angles of a triangle add to ____________

Example:

a) Two angles of a triangle are the same measure. The third angle is 30 degrees less than the first. Find the three angles.

b) The second angle of a triangle measures twice the first. The third angle is 30 degrees more than the second. Find the three angles.

**YOU TRY**

a) The second angle of a triangle is double the first. The third angle is 40 less than the first. Find the three angles.
B. PERIMETER
Another geometry problem involves perimeter or the distance around an object.

<table>
<thead>
<tr>
<th>Perimeter of a rectangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>The formula for the perimeter of a rectangle is given by ( P = 2w + 2l ) where ( w ) is the width and ( l ) is the length of the rectangle.</td>
</tr>
</tbody>
</table>

**MEDIA LESSON**
Perimeter problems (Duration 5:00)

*View the video lesson, take notes and complete the problems below.*

Formula for Perimeter of a rectangle: ________________________________

Width is the __________ side.

Example:

a) A rectangle is three times as long as it is wide. If the perimeter is 62 cm, what is the length?

b) The width of a rectangle is 6 cm less than the length. If the perimeter is 52 cm, what is the width?

**YOU TRY**

The perimeter of a rectangle is 44 cm. The length is 5 less than double the width. Find the dimensions.
EXERCISES

1) The second angle of a triangle is the same size as the first angle. The third angle is 12 degrees larger than the first angle. How large are the angles?

2) Two angles of a triangle are the same size. The third angle is 12 degrees smaller than the first angle. Find the measure the angles.

3) Two angles of a triangle are the same size. The third angle is 3 times as large as the first. How large are the angles?

4) The third angle of a triangle is the same size as the first. The second angle is 4 times the third. Find the measure of the angles.

5) The second angle of a triangle is 3 times as large as the first angle. The third angle is 30 degrees more than the first angle. Find the measure of the angles.

6) The second angle of a triangle is twice as large as the first. The measure of the third angle is 20 degrees greater than the first. How large are the angles?

7) The second angle of a triangle is three times as large as the first. The measure of the third angle is 40 degrees greater than that of the first angle. How large are the three angles?

8) The second angle of a triangle is five times as large as the first. The measure of the third angle is 12 degrees greater than that of the first angle. How large are the angles?

9) The second angle of a triangle is three times the first, and the third is 12 degrees less than twice the first. Find the measures of the angles.

10) The second angle of a triangle is four times the first and the third is 5 degrees more than twice the first. Find the measures of the angles.

11) The perimeter of a rectangle is 150 cm. The length is 15 cm greater than the width. Find the dimensions.

12) The perimeter of a rectangle is 304 cm. The length is 40 cm longer than the width. Find the length and width.

13) The perimeter of a rectangle is 152 meters. The width is 22 meters less than the length. Find the length and width.

14) The perimeter of a rectangle is 280 meters. The width is 26 meters less than the length. Find the length and width.

15) The perimeter of a college basketball court is 96 meters and the length is 14 meters more than the width. What are the dimensions?

Log on to Canvas to take the section quiz
SECTION 4.4: VALUE AND INTEREST PROBLEMS
A common type of word problems is value and interest problems.

A. VALUE PROBLEMS WITH COINS
We will use a common formula to model the word problems called $AVT$:

\[ Amount \cdot Value = Total \]

MEDIA LESSON
Value problems (Duration 5:00)

View the video lesson, take notes and complete the problems below.

Value table:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The equation always comes from the ________________________________

Example:

a) Brian has twice as many dimes as quarters. If the value of the coin is $4.95, how many of each does he have?

b) A child has three more nickels than dimes in her piggy bank. If she has $1.95 in the bank, how many of each does she have?

YOU TRY

a) In a child’s piggy bank are a mix of 11 quarters and dimes that has a total value of $1.85. How many coins of each does the child have?

b) A man has a collection of stamps made up of 5-cent stamps and 8-cent stamps. There are three times as many 8-cent stamps as 5-cent stamps. The total value of all the stamps is $3.48. How many of each stamp does the man have?
B. SIMPLE INTEREST PROBLEMS

MEDIA LESSON
Simple Interest (Duration 6:17)

View the video lesson, take notes and complete the problems below.

Interest represents a charge in money.

If you have a saving account, the __________________________________________________________
____________________________________________________________________________________.

If you have a loan, the __________________________________________________________________
____________________________________________________________________________________.

Simple Interest Formula

\[ I = \frac{P \times r \times t}{1} \]

Example:

a) If you invest $3,500 in saving account that pays 4% simple interest, how much interest will you earn after 3 years? What will the new balance be?

b) You borrow $6,000 from a loan shark. If you will owe $7,200 in 18 months, what would be the simple interest rate?

YOU TRY

a) Jesse invests $500 in a simple interest account at 5%. How much interest will he earn after 4 years?

b) Karla earned $180 of interest at 4% after she deposited $1500 in a saving account. How long did she leave her money in there?
C. VALUE/INTEREST PROBLEMS WITH 1 VARIABLE

For interest problems below, we can use a similar formula \( I = Prt \), but in this section, we will assume that the time is one year, i.e., \( t = 1 \) to get the formula below.

\[
\begin{align*}
Principal \cdot rate \cdot time &= Interest \\
Principal \cdot rate \cdot 1 &= Interest \\
Principal \cdot rate &= Interest
\end{align*}
\]

View the video lesson, take notes and complete the problems below.

Interest table:

\[\begin{array}{c|c|c|c}
\hline
& & & \\
\hline
\end{array}\]

The equation always comes from the _______________________________

Example:

a) Sophia invested $1900 in one account and $1500 in another account that paid 3% higher interest rate. After one year she had earned $113 in interest. At what rates did she invest?

<table>
<thead>
<tr>
<th>Principal</th>
<th>Rate</th>
<th>Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Carlos invested $2500 in one account and $1000 in another which paid 4% lower interest. At the end of a year, he had earned $345 in interest. At what rates did he invest?

<table>
<thead>
<tr>
<th>Principal</th>
<th>Rate</th>
<th>Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

YOU TRY

a) A woman invests $4,000 in two accounts: one at 6% interest, the other at 9% interest for one year. At the end of the year, she had earned $270 in interest. How much did she invest in each account?

<table>
<thead>
<tr>
<th>Principal</th>
<th>Rate</th>
<th>Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) John invests $5,000 in one account and $8,000 in another account paying 4% more in interest. He earned $1,230 in interest after one year. At what rates did he invest?

<table>
<thead>
<tr>
<th>Principal</th>
<th>Rate</th>
<th>Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
EXERCISES

1) A collection of dimes and quarters is worth $15.25. There are 103 coins in all. How many of each kind is there?

2) A collection of half dollars and nickels is worth $13.40. There are 34 coins in all. How many of each kind is there?

3) A purse contains $3.90 made up of dimes and quarters. If there are 21 coins in all, how many dimes and how many quarters were there?

4) A boy has $2.25 in nickels and dimes. If there are twice as many dimes as nickels, how many of each kind does he have?

5) $3.75 is made up of quarters and half dollars. If the number of quarters exceeds the number of half dollars by 3, how many coins of each denomination are there?

6) A collection of 27 coins consisting of nickels and dimes amounts to $2.25. How many coins of each kind are there?

7) $3.25 in dimes and nickels were distributed among 45 boys. If each received one coin, how many received dimes and how many received nickels?

8) A coin purse contains 18 coins in nickels and dimes. The coins have a total value of $1.15. Find the number of nickels and dimes in the coin purse.

9) The total value of dimes and quarters in a bank is $6.05. There are six more quarters than dimes. Find the number of each type of coin in the bank.

10) A child’s piggy bank contains 44 coins in quarters and dimes. The coins have a total value of $8.60. Find the number of quarters in the bank.

11) A coin bag contains nickels and dimes. The number of dimes is 10 less than twice the number of nickels. The total value of all the coins is $2.75. Find the number of each type of coin in the bank.

12) A total of 26 bills are in a cash box. Some of the bills are one dollar bills, and the rest are five dollar bills. The total amount of cash in the box is $50. Find the number of each type of bill in the cash box.

13) A bank teller cashed a check for $200 using twenty-dollar bills and ten-dollar bills. In all, 12 bills were handed to the customer. Find the number of twenty-dollar bills and the number of ten-dollar bills.

14) Find the simple interest rate for $2,000 at 6% for 2 years.

15) What is the balance of an account after 10 years with an initial $3,000 deposit at 3% interest?

16) Jerry got a $22,000 car loan with the simple interest rate at 4.25% for 60 months. How much would he pay to the bank in total when the loan term is over?

17) Lupita deposited $800 into a saving account 4 years ago. She has left her money alone since then. She has $896 in her account today. What is the interest rate of the account?

18) Elena deposited $2,000 in her savings account at 5%. How long would it take for her to earn $1,000 interest?

19) You want to borrow $10,000 to buy a car. The dealership has a promotion at 1.99% simple rate. How much do you have to pay in interest after 60 months?
20) Jessica makes $395 on interest in her savings account after 5 years at 2%. How much did she deposit into her account initially?

21) How long would it take Jose to earn $660 in interest after he deposited $1000 into an account with a 5.5% rate?

22) Jose invested $12,500 with Bank USA and $14,500 with Bank of Pacifica that offered 1% higher interest rate. He earned $3,385 in total interest after one year. At what rate did each bank offer?

23) Angela invested $2,500 in the Wealthy Future mutual fund plan and $6,500 in the Blue Chip plan that yields 2% more. The total interest after one year was $1,030. At what rate did each plan offer?

24) A college fund is invested with two different banks, $4,100 with Bank A and $5,900 at Bank B, which offered at 1.5% higher rate. The combined annual interest was $1,038.50. What was the interest rate at each bank?

25) Jason earned $256 interest last year on his investments. If $1,600 was invested at a certain rate of return and $2,400 was invested in a fund with a rate that was double the rate of the first fund, find the two rates of interest.

26) Natalia invested $3,500 in the AB High yield fund and $5,000 in the Emerging Market Core fund. The AB high yield rate is 2.5% higher than the Emerging Market rate. Natalia earned $385 interest from both accounts after one year. What was interest rate for each fund?

27) A person borrowed $6,500 at QuickCash and $8500 at CashNow. QuickCash charges 3% less than CashNow. After one year, he paid a total interest of $1,455. What was the simple interest rate each company charged him?

28) Kenny took out two different loans to buy a used car for his teenage daughter. He got $3000 at Prosperity Bank and $3000 from Trusty Bank, which charged him 1.5% higher rate. He paid $300 in simple interest to both banks after a year. At what rate did each bank charge him?

29) Yang owed two credit cards with balances of $7,500 and $3,500. The card with the $3,500 balance charged 1.4% more than the other. At the end of one year, he paid $797. What was the simple interest of each card?

30) Samantha earned $1,480 in interest last year on her investments. If $5,000 was invested in Aggressive portfolio with a 4% higher rate of return than $11,000 that was invested in Moderate portfolio. Find the return rate of each portfolio.

Log on to Canvas to take the section quiz
SECTION 4.5: UNIFORM MOTION PROBLEMS

Another common application of linear equations is uniform motion problems. When solving uniform motion problems, we use the relationship the distance formula

\[ \text{distance} = \text{rate} \cdot \text{time} \quad \text{(speed)} \]
or

\[ \text{rate} \cdot \text{time} = \text{distance} \]

A. DISTANCE = RATE X TIME

View the video lesson, take notes and complete the problems below.

Example: If you drive 65 miles per hour for 4 hours, how far have you traveled?

MEDIA LESSON
Distance problems – Find d (Duration 1:39)

Example: Google maps shows your trip will require you to drive 377 miles. If you average 58 miles per hour, how long will the drive take you?

MEDIA LESSON
Distance problems – Find t (Duration 2:28)

Example: If you drive for 7 hours and travel 385 miles, what is your rate?

MEDIA LESSON
Distance problems – Find r (Duration 2:51)
B. OPPOSITE DIRECTIONS

View the video lesson, take notes and complete the problems below.

Distance table:

The equation always comes from the ________________________________

Example:

a) Brian and Jennifer both leave the convention at the same time traveling in opposite directions. Brian drove 35 mph and Jennifer drove 50 mph. After how much time were they 340 miles apart?

b) Maria and Tristan are 126 miles apart biking towards each other. If Maria bikes 6 mph faster than Tristan and they meet after 3 hours, how fast did they each ride?

YOU TRY

a) Two joggers start from opposite ends of an 8-mile course running towards each other. One jogger is running at a rate of 4 miles per hour, and the other is running at a rate of 6 miles per hour. After how long will the joggers meet?

b) Bob and Fred start from the same point and walk in opposite directions. Bob walks 2 miles per hour faster than Fred. After 3 hours, they are 30 miles apart. How fast did each walk?

c) Two campers left their campsite by canoe and paddled downstream at an average speed of 12 mph. They turned around and paddled back upstream at an average rate of 4 mph. The total trip took 1 hour. After how much time did the campers turn around downstream?
C. CATCH-UP

View the video lesson, take notes and complete the problems below.

A head start: __________________ the head start to his/her __________________

Catch up:

Example:

a) Raquel left the party traveling 5mph. 4 hours later Nick left to catch up with her, traveling 7mph. How long will it take him to catch up?

b) Trey left on a trip traveling 20 mph. Julian left 2 hours later, traveling in the same direction at 30 mph. After how many hours does Julian pass Trey?

YOU TRY

a) Mike leaves his house traveling 2 miles per hour. Joy leaves 6 hours later to catch up with him traveling 8 miles per hour. How long will it take her to catch up with him?
D. TOTAL TIME

View the video lesson, take notes and complete the problems below.

Consider: Total time of 8...

When we have total time, for the first box we use _______ and for the second we use _____________.

Example:

a) Lupe rode into the forest at 10 mph, turned around and returned by the same route traveling 15 mph. If her trip took 5 hours, how long did she travel at each rate?

b) Ian went on a 230-mile trip. He started driving 45 mph. However, due to construction on the second leg of the trip, he had to slow down to 25 mph. If the trip took 6 hours, how long did he drive at each speed?

YOU TRY

a) On a 130-mile trip, a car traveled at an average speed of 55 mph and then reduced its speed to 40 mph for the remainder of the trip. The trip took 2.5 hours. For how long did the car travel 40 mph?
EXERCISES

1) A train traveled for 2 hours at a speed of 45 mph and 5 hours at 95 mph. What is the total miles that the train traveled?

2) The distance from LAX to San Francisco is 385 miles. If Gabriela drove at the average speed of 55mph, how long did it take her to arrive?

3) Sofia and her friends drove from Orange County to Las Vegas to celebrate her 21st birthday. It took them 4 hours to get there. The distance from Orange County to Vegas is 262 miles. What was their average speed?

4) It takes 6 hours to fly from John Wayne airport to JFK airport, New York that is 2454 air miles. What is the average speed of the plane?

5) Ana rode from Orange to Yosemite National Park with her bike group a total of 450 miles. It took them 45 hours in total. What was their average speed?

6) A is 60 miles from B. An automobile at A starts for B at the rate of 20 miles an hour while an automobile at B starts for A at the rate of 25 miles an hour. How long will it be before the automobiles meet?

7) Two trains travel toward each other from points which are 195 miles apart. They travel at rate of 25 and 40 miles an hour, respectively. If they start traveling at the same time, how long before the trains will meet?

8) A passenger and a freight train start toward each other at the same time from two points 300 miles apart. If the rate of the passenger train exceeds the rate of the freight train by 15 miles per hour, and they meet after 4 hours, what are the rates of each train?

9) A man having ten hours at his disposal made an excursion, riding out at the rate of 10 miles an hour and returning on foot at the rate of 3 miles an hour. Find the distance he rode.

10) A boy rides away from home in an automobile at the rate of 28 miles an hour and walks back at the rate of 4 miles an hour. The round trip requires 2 hours. How far does he ride in the automobile?

11) A family drove to a resort at an average speed of 30 mph and later returned over the same road at an average speed of 50 mph. Find the distance to the resort if the total driving time was 8 hours.

12) Annie, who travels 4 miles an hour starts from a certain place 2 hours in advance of Brandie, who travels 5 miles an hour in the same direction. How many hours must Brandie travel to overtake Annie?

13) A motorboat leaves a harbor and travels at an average speed of 8 mph to a small island. Two hours later a cabin cruiser leaves the same harbor and travels at an average speed of 16 mph to the same
island. In how many hours after the cabin cruiser leaves will the cabin cruiser be alongside the
motorboat?

14) A car traveling at 48 mph overtakes a cyclist who, riding at 12 mph, has had a 3-hour head start. How far from the starting point does the car overtake the cyclist?

15) Two men are traveling in opposite directions at the rate of 20 and 30 miles per hour at the same time and from the same place. In how many hours will they be 300 miles apart?

16) A motor boat leaves a harbor and travels at an average speed of 18 mph to an island. The average speed on the return trip was 12 mph. How far was the island from the harbor if the total trip took 5 hours?

17) A jet plane traveling at 570 mph overtakes a propeller-driven plane that has had a 2-hour head start. The propeller-driven plane is traveling at 190 mph. How far from the starting point does the jet overtake the propeller-driven plane?

18) As part of flight training, a student pilot was required to fly to an airport and then return. The average speed on the way to the airport was 100 mph, and the average speed returning was 150 mph. Find the distance between the two airports if the total flight time was 5 hours.

19) A car traveling at 56 mph overtakes a cyclist who, riding at 14 mph, has had a 3-hour head start. How far from the starting point does the car overtake the cyclist?

20) A bus traveling at a rate of 60 mph overtakes a car traveling at a rate of 45 mph. If the car had a 1-hour head start, how far from the starting point does the bus overtake the car?

21) A truck leaves a depot at 11 a.m. and travels at a speed of 45 mph. At noon, a van leaves the same place and travels the same route at a speed of 65 mph. At what time does the van overtake the truck?

22) Three campers left their campsite by canoe and paddled downstream at an average rate of 10 mph. They then turned around and paddled back upstream at an average rate of 5 mph to return to their campsite. How long did it take the campers to canoe downstream if the total trip took 1 hour?

Log on to Canvas to take the section quiz
SECTION 4.6 MIXTURE PROBLEMS

The last common type of word problems in this chapter is mixture problems. Mixture problems are problems where we mix two different categories resulting in one new category. E.g., we can mix cashews with almonds and create a mix of nuts, or we can mix alcohol with water and create a new solution, etc. We will use a common formula to model the word problems called ACT:

\[
\text{Amount} \cdot \text{Concentration} = \text{Total Cost} \quad \text{or} \quad \text{Final Cost}
\]

<table>
<thead>
<tr>
<th>MEDIA LESSON</th>
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<tbody>
<tr>
<td>Mixture problems – Known starting amount (Duration 5:00)</td>
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View the video lesson, take notes and complete the problems below.

Mixture table:

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The equation always comes from the _______________________

Example:

a) A storeowner wants to mix chocolate and nuts to make a new candy. How many pounds of chocolate costing $8.50/lb. should be mixed with 25 pounds of nuts that cost $2.50/lb. to make a mixture worth $4.33/lb.?

b) You need a 55% alcohol solution. On hand, you have 600 mL of 10% alcohol mixture. How much of the 95% mixture should you add to obtain your desired solution?

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YOU TRY

a) A chemist has 70 mL of a 50% methane solution. How much of an 80% solution must she add so the final solution is 60% methane?

b) A café sells a two-origin coffee for $2.50. The café has a single-origin coffee that costs $3.00 for 40 ml. How much of a second coffee that costs $1.50 should the café mix with the first to create the two-origin coffee?
EXERCISES

1) How much antifreeze should be added to 5 quarts of a 30% mixture of antifreeze to make a solution that is 50% antifreeze?

2) A tank contains 8000 liters of a solution that is 40% acid. How much water should be added to make a solution that is 30% acid? Note that water has 0% of acid.

3) Of 12 pounds of salt water, 10% is salt. In another mixture, 3% is salt. How many pounds of the second should be added to the first to get a mixture of 5% salt?

4) How many pounds of a 4% solution of borax must be added to 24 pounds of a 12% solution of borax to obtain a 10% solution of borax?

5) A 100 lb. bag of animal feed is 40% oats. How many pounds of oats must be added to this feed to produce a mixture which is 50% oats?

6) A 20-ounce alloy of platinum that costs $220 per ounce is mixed with an alloy that costs $400 per ounce. How many ounces of the $400 alloy should be used to make an alloy that costs $300 per ounce?

7) How many pounds of tea that cost $4.20 per pound must be mixed with 12 pounds of tea that cost $2.25 per pound to make a mixture that costs $3.40 per pound?

8) How many liters of a solvent that costs $80 per liter must be mixed with 6 liters of a solvent that costs $25 per liter to make a solvent that costs $36 per liter?

9) How many kilograms of hard candy that cost $7.50 per kilogram must be mixed with 24 kilograms of jelly beans that cost $3.25 per kilogram to make a mixture that sells for $4.50 per kilogram?

10) How many kilograms of soil supplement that costs $7 per kilogram must be mixed with 20 kilograms of aluminum nitrate that costs $3.50 per kilogram to make a fertilizer that costs $4.50 per kilogram?

11) How many pounds of lima beans that costs 90¢ per pound must be mixed with 16 pounds of corn that costs 50 cents per pound to make a mixture of vegetables that costs 65 cents per pound?

12) How many liters of a blue dye that costs $1.60 per liter must be mixed with 18 liters of another concentration that costs $2.50 per liter to make a mixture that costs $1.90 per liter?

13) How many ounces of dried apricots must be added to 18 ounces of a snack mix that contains 20% dried apricots to make a mixture that is 25% dried apricots?

14) How many pounds of coffee that is 40% Java beans must be mixed with 80 pounds of coffee that is 30% Java beans to make a coffee blend that is 32% Java beans?

15) The manager of a garden shop mixes grass seed that is 60% rye grass with 70 pounds of grass seeds that is 80% rye grass to make a mixture that is 74% rye grass. How much of the 60% mixture is used?

Log on to Canvas to take the section quiz
CHAPTER REVIEW

KEY TERMS AND CONCEPTS

Look for the following terms and concepts as you work through the workbook. In the space below, explain the meaning of each of these concepts and terms in your own words. Provide examples that are not identical to those in the text or in the media lesson.

<table>
<thead>
<tr>
<th>Term</th>
<th>Explanation</th>
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<td>Consecutive integers</td>
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<td>Mark-up formula</td>
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<td>Discount formula</td>
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<td>Sum of Angles in a Triangle</td>
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<td>Perimeter of a rectangle formula</td>
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<td>AVT formula</td>
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<td>Simple Interest formula</td>
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<td>Distance formula</td>
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<tr>
<td>Mixture formula</td>
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1) When 10 is subtracted from 4 times a certain number, the result is 26. What is the number?

2) Forty more than two times a number is the same as eight less than ten times the number. What is the number?

3) The sum of three consecutive integers is 93. What are the integers?

4) The sum of two consecutive even integers is 126. What are the integers?
5) Find three consecutive odd integers such that the sum is 63.

6) A jewelry storeowner bought a diamond in St. Thomas. He then sold it in the United States for $4,000. If he sold it at a 80% mark-up, what was his original cost for the diamond in St. Thomas? Round your answer to the nearest cent.

7) Mickey wants to buy her first new car, but she only has $25,000 to spend. She has her eye on a $20,000 hybrid. However, taxes, license and registration tack on an extra 9.5% of the purchase price. What would her actual cost be? Can she afford this car?

8) Jay found a rare book at an estate sale and purchased it for $400. He then sold it online for $800. What was his percent markup?

9) Bella’s clothing store pays $9.25 per pair of earrings. The store mark-ups 200%. How much is the selling price for a pair of earrings? Round your answer to the nearest cent.

10) The Fancy steakhouse marks-up their steaks 400%. Each piece of premium steak cost $15.00 at wholesale price. What is the selling price of a piece of steak?

11) The second angle of a triangle is the same size as the first angle. The third angle is 10 degrees larger than the first angle. How large are the angles?

12) Two angles of a triangle are the same size. The third angle is 4 times as large as the first. How large are the angles?

13) The second angle of a triangle is 2 times as large as the first angle. The third angle is 25 degrees more than the first angle. Find the measure of the angles.

14) The second angle of a triangle is four times the first and the third is 10 degrees more than twice the first. Find the measure of the angles.

15) The perimeter of a college basketball court is 100 meters and the length is 10 meters more than the width. What are the dimensions?

16) A collection of dimes and quarters is worth $10.35. There are 90 coins in all. How many of each is there? Let x be the number of dimes.

17) $7.00 is made up of quarters and half dollars. If the number of quarters exceeds the number of half dollars by $2, how many coins of each denomination are there? Let x be the number of half dollars.

18) A child’s piggy bank contains 43 coins in quarters and dimes. The coins have a total value of $10.00. Find the number of quarters in the bank. Let x be the number of quarters.
19) What is the balance of an account after 20 years with an initial $4,000 deposit at 4% interest?

20) Jessica makes $595 on interest in her saving account after 5 years at 3%. How much did she deposit into her account initially?

21) A train traveled for 2 hours at a speed of 35 mph and 4 hours at 105 mph. What is the total miles that the train traveled?

22) It takes 5 hours to fly from John Wayne airport to JFK airport, New York that is 3400 air miles. What is the average speed of the plane?

23) A man having ten hours at his disposal made an excursion, riding out at the rate of 15 miles an hour and returning on foot at the rate of 5 miles an hour. Find the distance he rode.

24) A car traveling at 40 mph overtakes a cyclist who, riding at 10 mph, has had a 4-hour head start. How far from the starting point does the car overtake the cyclist?

25) A car traveling at 65 mph overtakes a cyclist who, riding at 14 mph, has had a 4-hour head start. How far from the starting point does the car overtake the cyclist?

26) Of 10 pounds of salt water, 5% is salt. In another mixture, 8% is salt. How many pounds of the second should be added to the first to get a mixture of 6% salt? Let x represent the amount of salt in the 8% mixture.

27) How many pounds of tea that cost $6.00 per pound must be mixed with 10 pounds of tea that cost $2.00 per pound to make a mixture that costs $4.00 per pound?

28) How many kilograms of hard candy that cost $5.00 per kilogram must be mixed with 20 kilograms of jelly beans that cost $2.25 per kilogram to make a mixture that sells for $3.75 per kilogram?

29) How many pounds of lima beans that cost $0.80 per pound must be mixed with 15 pounds of corn that cost $0.40 per pound to make a mixture of vegetables that costs $0.50 per pound?

30) How many ounces of dried apricots must be added to 10 ounces of a snack mix that contains 30% dried apricots to make a mixture that is 20% dried apricots?