CHAPTER 2: INTRODUCTION TO VARIABLES AND PROPERTIES OF ALGEBRA

Chapter Objectives

By the end of this chapter, students should be able to:

✓ Interpret different meanings of variables
✓ Evaluate algebraic expressions
✓ Writing algebraic expressions
✓ Identify properties of algebra
  • Commutative
  • Associative
  • Identity
  • Inverse
  • Distributive

Contents

CHAPTER 2: INTRODUCTION TO VARIABLES AND PROPERTIES OF ALGEBRA ................................................................. 89

SECTION 2.1 INTRODUCTION TO VARIABLES ............................................................................................... 90

A. WHAT IS A VARIABLE? ......................................................................................................................... 90
B. MEANING OF A VARIABLE IN MATHEMATICS ............................................................................... 92
C. VARIABLE EXPRESSIONS .................................................................................................................. 93
D. WRITING ALGEBRAIC EXPRESSIONS ............................................................................................... 94
E. EVALUATE ALGEBRAIC EXPRESSIONS .......................................................................................... 98
F. LIKE TERMS AND COMBINE LIKE TERMS .................................................................................... 100

EXERCISES ............................................................................................................................................. 101

SECTION 2.2 PROPERTIES OF ALGEBRA ................................................................................................. 103

EXERCISES ............................................................................................................................................. 106

CHAPTER REVIEW .................................................................................................................................... 108
SECTION 2.1 INTRODUCTION TO VARIABLES

A. WHAT IS A VARIABLE?

When someone is having trouble with algebra, they may say, “I don’t speak math!” While this may seem weird to you, it is a true statement. Math, like English, French, Spanish, or Arabic, is a like a language that you must learn in order to be successful. In order to understand math, you must practice the language.

Action words, like run, jump, or drive, are called verbs. In mathematics, operations are like verbs because they involve doing something. Some operations are familiar, such as addition, multiplication, subtraction, or division.

Operations can also be much more complex like a raising to an exponent or taking a square root.

A variable is one of the most important concepts of mathematics, without variables we would not get far in this study. A variable is a symbol, usually an English letter, written to replace an unknown or changing quantity.

<table>
<thead>
<tr>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A variable</strong>, usually represented by a letter or symbol, can be defined as:</td>
</tr>
<tr>
<td>• A quantity that may change within the context of a mathematical problem.</td>
</tr>
<tr>
<td>• A placeholder for a specific value.</td>
</tr>
<tr>
<td>An algebraic expression is a mathematical statement that can contain numbers, variables, and operations (addition, subtraction, multiplication, division, etc...).</td>
</tr>
</tbody>
</table>

**MEDIA LESSON**

Introduction to variables and variable expressions (Duration 7:54)

View the video lesson, take notes and complete the problems below.

**Definition:**

• A ____________________ is a ________________ that represents an ________________________.

  a) How many hours will you study tomorrow?

  _____________________________________________________________

  _____________________________________________________________

  b) How much will you pay to have your car repaired?

  _____________________________________________________________

  _____________________________________________________________

• A__________________________ consists of ________________________, _______________, ________________, and ________________ like ________________, ________________, ________________, and ________________.

• Often ___________________________ and ___________________________ are used ___________________________. The difference is a ___________________________ contain a variable while the ___________________________.
• Writing variable expressions
  
a) It costs $9 per adult and $5 per child to go to the movies.
  Variable expression for the cost of a group go to the movies: ______________________

  __________________________________________

  __________________________________________

  __________________________________________
b) It costs $30 per day to rent a car plus $0.10 per mile.
  Variable expression for the total rental cost: ______________________

  __________________________________________

  __________________________________________

  __________________________________________

Evaluating variable expressions

c) Evaluate $5x + 7$ when $x = 6$

d) Evaluate $4m - 3n$ when $m = 9$ and $n = 7$

e) Evaluate $p^2 - 3q + 7$ when $p = 11, q = 8$

f) Evaluate $\frac{36}{d} + 7e - 9f$ when $d = 9, e = 3, \text{ and } f = 1$

MEDIA LESSON
Why all the letters in algebra? (Duration 3:03)

View the video lesson, take notes and complete the problems below.

What question do students ask a lot when they start Algebra?

_____________________________________________________________________________________

What are other things besides letters that can be used as a variable?

_____________________________________________________________________________________
B. MEANING OF A VARIABLE IN MATHEMATICS

View the video lesson, take notes and complete the problems below.

Definitions of a variable

1) **Define variable as a changing value**
   A variable is a letter that represents ___________ or ___________ that may _______________ within the context of a mathematical problem.
   
   **Example:**
   __________________________________________________________________________
   ______________________________________________________________________________

2) **Define variable as a placeholder**
   A variable is a letter that represents ___________ or ___________ that will remain ___________ based on the confines of the mathematical problem.
   
   **Example:**
   __________________________________________________________________________
   ______________________________________________________________________________

Example: Determine if the description describes a changing value (CV) or a placeholder (P) then determine a possible variable.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Changing Value (CV) or Placeholder (P)</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>The elevation of Mount Humphrey's</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The water level of a pool as it is being filled</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The number of calories consumed throughout the day</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The monthly car payment with a fixed interest loan</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The amount of gas consumed by your car as you drive</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The cost of a new textbook for a specific class from the bookstore at the beginning of the semester</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**YOU TRY**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Changing Value (CV) or Placeholder (P)</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>The number of cars in the parking lot at this moment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The number of cars pass by an intersection throughout the day</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The altitude of an airplane during a trip</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The temperature in the Dead Valley on at midnight on January 1st in 1905</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The balance of your checking account today</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Why aren't we using the multiplication sign?

Why the multiplication sign “x” is not being used much when we get to algebra?

Why is ‘x’ the symbol for an unknown?

Why is it that \( x \) is the unknown?

C. VARIABLE EXPRESSIONS

In mathematics, especially in algebra, we look for patterns in the numbers that we see. Using mathematical verbs and variables studied in previous lessons, expressions can be written to describe a pattern.

Definition

An algebraic expression is a mathematical phrase combining numbers and/or variables using mathematical operations.

An algebraic expression consists of coefficients, variables, and terms. Given an algebraic expression, a

- coefficient is the number in front of the variable.
- variable is a letter representing any number.
- term is a product of a coefficient and variable(s).
- constant: a number with no variable attached. A term whose value never changes.

For example, \( t, 2x, 3st, 7x^2, 5ab^3c \) are all examples of terms because each is a product of a coefficient and variable(s).

Terms: ______________________________________________________________________________.

Constant Term: _________________________________________________________________________.

Example 1: Consider the algebraic expression \( 4x^5 + 3x^4 - 22x^2 - x + 17 \)

a) List the terms: _______________________________________________________________________

b) Identify the constant term: _____________________________________________________________

Factors: _______________________________________________________________________________.
Example 2: Complete the table below.

<table>
<thead>
<tr>
<th></th>
<th>(-4m)</th>
<th>(-x)</th>
<th>(\frac{1}{2}bh)</th>
<th>(\frac{2r}{5})</th>
</tr>
</thead>
<tbody>
<tr>
<td>List the Factors</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Identify the Coefficient</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Example 3: Consider the algebraic expression \(5y^4 - 8y^3 + y^2 - \frac{y}{4} - 7\).

a) How many terms are there? ____________
b) Identify the constant term. ____________
c) What is the coefficient of the first term? ____________
d) What is the coefficient of the second term? ____________
e) What is the coefficient of the third term? ____________
f) List the factors of the fourth term._____________________________________

YOU TRY

Consider the algebraic expression \(2m^3 + m^2 - 2m - 8\).

a) How many terms are there? ____________
b) Identify the constant term. ____________
c) What is the coefficient of the first term? ____________
d) What is the coefficient of the second term? ____________
e) List the factors of the third term. ________________

D. WRITING ALGEBRAIC EXPRESSIONS

View the video lesson, take notes and complete the problems below.

Example 1: Juan is 6 inches taller than Niko. Let \(N\) represent Niko’s height in inches. Write an algebraic expression to represent Juan’s height.

Niko’s height: _________________________________
Juan’s height: _________________________________

Example 2: Juan is 6 inches taller than Niko. Let \(J\) represent Juan’s height in inches. Write an algebraic expression to represent Niko’s height.

Niko’s height: _________________________________
Juan’s height: _________________________________
Example 3: Suppose sales tax in your town is currently 9.8%. Write an algebraic expression representing the sales tax for an item that costs $D$ dollars.

Cost: ________________________________

Sales tax: ________________________________

Example 4: You started this year with $362 saved and you continue to save an additional $30 per month. Write an algebraic expression to represent the total amount saved after $m$ months.

Number of months: ________________________________

Total amount saved: ________________________________

Example 5: Movie tickets cost $8 for adults and $5.50 for children. Write an algebraic expression to represent the total cost for $A$ adults and $C$ children to go to a movie.

Number of adults: ________________________________

Number of children: ________________________________

Total Cost: ________________________________

YOU TRY

Complete the following problems. Show all steps as in the media examples.

a) There are about 80 calories in one chocolate chip cookie. If we let $n$ be the number of chocolate chip cookies eaten, write an algebraic expression for the number of calories consumed.

Number of cookies: ________________________________

Number of calories consumed: ________________________________

b) Brendan recently hired a contractor to do some necessary repair work. The contractor gave a quote of $450 for materials and supplies plus $38 an hour for labor. Write an algebraic expression to represent the total cost for the repairs if the contractor works for $h$ hours.

Number of hours worked: ________________________________

Total cost: ________________________________

c) A concession stand charges $3.50 for a slice of pizza and $1.50 for a soda. Write an algebraic expression to represent the total cost for $P$ slices of pizza and $S$ sodas.

Number of slices of pizza: ________________________________

Number of sodas: ________________________________

Total cost: ________________________________
View the video lesson, take notes and complete the problems below.

**Example 1**: Tell the story of $x$ in each of the following expressions.

a) $x - 5$  

b) $5 - x$  

c) $2x$  

d) $x^2$

**Example 2**: Tell the story of $x$ in each of the following expressions.

a) $2x + 4$  

b) $2(x + 4)$  

c) $5(x - 3)^2 - 2$

**Example 3**: Write an algebraic expression that summarizes the stories below.

a) Step 1: Add 3 to $x$  
   Step 2: Divide by 2  

b) Step 1: Divide $x$ by 2  
   Step 2: Add 3

**Example 4**: Write an algebraic expression that summarizes the stories below.

Step 1: Subtract $x$ from 7  
Step 2: Raise to the third power  
Step 3: Multiply by 3  
Step 4: Add 1
Below is a table of common English words converted into a mathematical expression. You can use this table to assist in translating expressions.

<table>
<thead>
<tr>
<th>Operation</th>
<th>Words</th>
<th>Example</th>
<th>Translation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Addition</strong></td>
<td>Added to</td>
<td>4 added to ( n )</td>
<td>( n + 4 )</td>
</tr>
<tr>
<td></td>
<td>More than</td>
<td>2 more than ( y )</td>
<td>( y + 2 )</td>
</tr>
<tr>
<td></td>
<td>The sum of</td>
<td>The sum of ( r ) and ( s )</td>
<td>( r + s )</td>
</tr>
<tr>
<td></td>
<td>Increased by</td>
<td>( m ) increased by 6</td>
<td>( m + 6 )</td>
</tr>
<tr>
<td></td>
<td>The total of</td>
<td>The total of 8 and ( x )</td>
<td>( 8 + x )</td>
</tr>
<tr>
<td></td>
<td>Plus</td>
<td>( c ) plus 2</td>
<td>( c + 2 )</td>
</tr>
<tr>
<td><strong>Subtraction</strong></td>
<td>Minus</td>
<td>( x ) minus 1</td>
<td>( x - 1 )</td>
</tr>
<tr>
<td></td>
<td>Less than</td>
<td>5 less than ( y )</td>
<td>( y - 5 )</td>
</tr>
<tr>
<td></td>
<td>Less</td>
<td>4 less ( r )</td>
<td>( 4 - r )</td>
</tr>
<tr>
<td></td>
<td>Subtracted from</td>
<td>3 subtracted from ( t )</td>
<td>( t - 3 )</td>
</tr>
<tr>
<td></td>
<td>Decreased by</td>
<td>( m ) decreased by 10</td>
<td>( m - 10 )</td>
</tr>
<tr>
<td></td>
<td>The difference between</td>
<td>The difference between ( x ) and ( y )</td>
<td>( x - y )</td>
</tr>
<tr>
<td><strong>Multiplication</strong></td>
<td>Times</td>
<td>12 times ( x )</td>
<td>( 12 \cdot x )</td>
</tr>
<tr>
<td></td>
<td>Of</td>
<td>One-third of ( v )</td>
<td>( \frac{1}{3} v )</td>
</tr>
<tr>
<td></td>
<td>The product of</td>
<td>The product of ( n ) and ( k )</td>
<td>( nk ) or ( n \cdot k )</td>
</tr>
<tr>
<td></td>
<td>Multiplied by</td>
<td>( y ) multiplied by 3</td>
<td>( 3y )</td>
</tr>
<tr>
<td></td>
<td>Twice</td>
<td>Twice ( d )</td>
<td>( 2d ) or ( 2 \cdot d )</td>
</tr>
<tr>
<td><strong>Division</strong></td>
<td>Divided by</td>
<td>( n ) divided by 4</td>
<td>( \frac{n}{4} )</td>
</tr>
<tr>
<td></td>
<td>The quotient of</td>
<td>The quotient of ( t ) and ( x )</td>
<td>( \frac{t}{x} )</td>
</tr>
<tr>
<td></td>
<td>The ratio of</td>
<td>The ratio of ( x ) to ( p )</td>
<td>( \frac{x}{p} )</td>
</tr>
<tr>
<td></td>
<td>Per</td>
<td>2 per ( b )</td>
<td>( \frac{2}{b} )</td>
</tr>
<tr>
<td><strong>Power</strong></td>
<td>The square of</td>
<td>The square of ( y )</td>
<td>( y^2 )</td>
</tr>
<tr>
<td></td>
<td>The cube of</td>
<td>The cube of ( k )</td>
<td>( k^3 )</td>
</tr>
<tr>
<td><strong>Equals</strong></td>
<td>Is</td>
<td>Equal</td>
<td>Equal</td>
</tr>
<tr>
<td></td>
<td>Gives</td>
<td>Is equal to</td>
<td>Is equivalent to</td>
</tr>
<tr>
<td></td>
<td>Yields</td>
<td>Results in</td>
<td>was</td>
</tr>
</tbody>
</table>
YOU TRY

Complete the following problems.

a) Tell the story of \( x \) in the expression \( \frac{x-3}{5} \).

b) Write an algebraic expression that summarizes the story below:

Step 1: Multiply \( x \) by 2
Step 2: Add 5
Step 3: Raise to the second power.

View the video lesson, take notes and complete the problems below.

What algebraic expression did Sal start with when he discussed why the abstraction of mathematics is so fundamental?

____________________________________________________________________________________

E. EVALUATE ALGEBRAIC EXPRESSIONS

View the video lesson, take notes and complete the problems below

To evaluate a algebraic or variable expression:

1. ________________________________________________________________________________
2. ________________________________________________________________________________

PEMDAS

P: ________________________________________________________________________________
E: ________________________________________________________________________________
MD: ________________________________________________________________________________
AS: ________________________________________________________________________________
Example 1: Find the value of each expression when \( w = 2 \). Simplify your answers.

\[
\begin{align*}
\text{a)} & \quad w - 6 & \quad \text{b)} & \quad 6 - w & \quad \text{c)} & \quad 5w - 3 \\
\text{d)} & \quad w^3 & \quad \text{e)} & \quad 3w^2 & \quad \text{f)} & \quad (3w)^2 \\
\text{g)} & \quad \frac{4}{5w} & \quad \text{h)} & \quad \frac{5w}{4} & \quad \text{i)} & \quad 3^w
\end{align*}
\]

Example 2: Evaluate \( ab + c \) given \( a = -5, b = 7, \) and \( c = -3. \)

Example 3: Evaluate \( a^2 - b^2 \) given \( a = -5 \) and \( b = -3. \)

Example 4: A local window washing company charges $11.92 for each window plus a reservation fee of $7.

a) Write an algebraic expression to represent the total cost from the window washing company for washing \( w \) windows.

b) Use this expression to determine the total cost for washing 17 windows.

YOU TRY

Given \( a = 5, b = -1, c = 2, \) evaluate the expressions below. Show all steps.

\[
\begin{align*}
\text{a)} & \quad \text{Evaluate } b^2 - 4ac. & \quad \text{b)} & \quad \text{Evaluate } 2a - 5b + 7c.
\end{align*}
\]
F. LIKE TERMS AND COMBINE LIKE TERMS

**Definition**

Two terms are **like terms** if the base variable(s) and exponent on each variable are identical.

For example, $3x^2y$ and $-7x^2y$ are like terms because they both contain the same base variables, $x$ and $y$, and the exponents on $x$ (the $x$ is squared on both terms) and $y$ are the same.

**Combining like terms:** If two terms are like terms, we add (or subtract) the **coefficients**, then keep the variables (and exponents on the corresponding variable) the same.

---

**MEDIA LESSON**

**Like terms and combine like terms** (Duration 6:30)

*View the video lesson, take notes and complete the problems below.*

**Example 1:** Identify the like terms in each of the following expressions.

\[
3a - 6a + 10a - a \quad 5x - 10y + 6z - 3x \quad 7n + 3n^2 - 2n^3 + 8n^2 + n - n^3
\]

**Example 2:** Combine the like terms.

\[
3a - 6a + 10a - a \quad 5x - 10y + 6z - 3x \quad 7n + 3n^2 - 2n^3 + 8n^2 + n - n^3
\]

**YOU TRY**

Combine the like terms.

a) \(3x - 4x + x - 8x\)  
b) \(-5 + 2a^2 - 4a + a^2 + 7\)
EXERCISES

Tell the story of $x$ in each of the following expressions.

1) $x - 11$
2) $x + 5$
3) $5x$
4) $x^5$
5) $x^3$
6) $2 - x$
7) $2x - 3$
8) $8x^2$
9) $(2x)^2$
10) $7 - 2x$
11) $5(7 - x)^3$
12) $\left(\frac{3x - 3}{5}\right)^3$

Write an algebraic expression that summarizes the stories below.

13) Step 1: Add 8 to $x$
    Step 2: Raise to the third power
14) Step 1: Divide $x$ by 8
    Step 2: Subtract 5
15) Step 1: Subtract 3 from $x$
    Step 2: Multiply by 7
16) Step 1: Multiply $x$ by 10
    Step 2: Raise to the 3rd power
    Step 3: Multiply by 2
17) Step 1: Add 5 to $x$
    Step 2: Divide by 2
    Step 3: Raise to the second power
    Step 4: Add 8
18) Step 1: Raise $x$ to the second power
    Step 2: Multiply by 5
    Step 3: Subtract from 9
19) Step 1: Subtract $x$ from 2
    Step 2: Multiply by $-8$
    Step 3: Raise to the third power
    Step 4: Add 1
    Step 5: Divide by 3
20) Step 1: Multiply $x$ by $-4$
    Step 2: Add 9
    Step 3: Divide by 2
    Step 4: Raise to the fifth power

Find the value of each expression when $b = -8$. Simplify your answers.

21) $b - 11$
22) $b + 5$
23) $5b$
24) $b^2$
25) $b^3$
26) $2 - b$

Evaluate each of the following given $q = 10$.

27) $2q - 3$
28) $8q^2$
29) $(2q)^2$
30) $\frac{4}{7q}$
31) $7 - 2q$
32) $2^q$
Evaluate the following expressions for the given values. Simplify your answers.

33) $\frac{-b}{2a}$ for $a = 6, b = 4$

34) $\frac{4x-8}{5+x}$ for $x = 3$

35) $\frac{3}{5}ab$ for $a = 8, b = 1 \frac{2}{3}$

36) $3x^2 + 2x - 1$ for $x = -1$

37) $x^2 - y^2$ for $x = -3, y = -2$

38) $2x - 7y$ for $x = 5, y = 3$

39) Shea bought $c$ candy bars for $1.50 each. Write an algebraic expression for the total amount Shea spent.

40) Suppose sales tax in your town is currently 9%. Write an algebraic expression representing the sales tax for an item that costs $d$ dollars.

41) Ben bought $m$ movie tickets for $8.50 each and $p$ bags of popcorn for $3.50 each.
   a) Write an algebraic expression for the total amount Ben spent.
   b) Use this expression to determine the amount Ben will spend if he buys 6 movie tickets and 4 bags of popcorn.

42) Noelle is 5 inches shorter than Amy. Amy is $A$ inches tall. Write an algebraic expression for Noelle's height.

43) Jamaal studied $h$ hours for a big test. Karla studied one fourth as long. Write an algebraic expression for the length of time that Karla studied.

44) A caterer charges a delivery fee of $45 plus $6.50 per guest.
   a) Write an algebraic expression to represent the total catering cost if $g$ guests attend the reception.
   b) Use the expression to determine the amount of a company luncheon of 50 guests.

45) Tickets to the museum cost $18 for adults and $12.50 for children.
   a) Write an algebraic expression to represent the cost for $a$ adults and $c$ children to visit the museum.
   b) Use this expression to determine the cost for 4 adults and 6 children to attend the museum.

46) Consider the algebraic expression $5n^8 - n^5 + n^2 + \frac{n}{8} - 2$
   a) How many terms are there? __________
   b) Identify the constant term. _______________
   c) What is the coefficient of the first term? _________
   d) What is the coefficient of the second term? ______________
   e) What is the coefficient of the third term? ______________
   f) What is the coefficient of the fourth term? ______________

Combine the like terms.

47) $3d - 5d + d - 7d$

48) $3x^2 + 3x^3 - 9x^2 + x - x^3$

49) $a - 2b + 4a + b - (-2b)$

50) $3x - 7y + 9x - 5y - 7$

Log on to Canvas to take the section quiz
### Properties of Algebra

Properties of real numbers are the basic rules when we work with expressions in Algebra.

<table>
<thead>
<tr>
<th></th>
<th>Addition</th>
<th>Multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Identity</strong></td>
<td>$a + 0 = a$</td>
<td>$a \cdot 1 = a$</td>
</tr>
<tr>
<td>Hint: the number</td>
<td>Any number plus zero is the same number.</td>
<td>Any number multiplies by one is the same number.</td>
</tr>
<tr>
<td>stays true to its</td>
<td>$7 + 0 = 7$</td>
<td>$7 \cdot 1 = 7$</td>
</tr>
<tr>
<td>“identity”.</td>
<td>$x + 0 = x$</td>
<td>$x \cdot 1 = x$</td>
</tr>
<tr>
<td></td>
<td>$\star + 0 = \star$</td>
<td>$\star \cdot 1 = \star$</td>
</tr>
<tr>
<td><strong>Inverse</strong></td>
<td>$a + (−a) = 0$</td>
<td><strong>Multiplicative inverse</strong></td>
</tr>
<tr>
<td>Hint: “Inverse” →</td>
<td>Any number plus its opposite equals zero.</td>
<td>$a \cdot \frac{1}{a} = 1$</td>
</tr>
<tr>
<td>reverse → undo</td>
<td>$2 + (−2) = 0$</td>
<td>if $a$ is not zero</td>
</tr>
<tr>
<td></td>
<td>$x + (−x) = 0$</td>
<td>$\frac{1}{3} \cdot 3 = 1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{1}{x} \cdot x = 1$</td>
</tr>
<tr>
<td><strong>Commutative</strong></td>
<td>$a + b = b + a$</td>
<td>$a \cdot b = b \cdot a$</td>
</tr>
<tr>
<td>Hint: “Commute” →</td>
<td>You can add in any order.</td>
<td>You can multiply in any order.</td>
</tr>
<tr>
<td>move → switch places</td>
<td>$2 + 3 = 3 + 2$</td>
<td>$2 \cdot 3 = 3 \cdot 2$</td>
</tr>
<tr>
<td></td>
<td>$x + 5 = 5 + x$</td>
<td>$x \cdot 2 = 2 \cdot x$</td>
</tr>
<tr>
<td></td>
<td>$☺ + \star = \star + ☺$</td>
<td>$☺ \cdot \star = \star \cdot ☺$</td>
</tr>
<tr>
<td><strong>Associative</strong></td>
<td>$(a + b) + c = a + (b + c)$</td>
<td>$(a \cdot b) \cdot c = a \cdot (b \cdot c)$</td>
</tr>
<tr>
<td>Hint: “associate” →</td>
<td>When you add, you can group in any combination.</td>
<td>When you multiply, you can group in any combination.</td>
</tr>
<tr>
<td>different groups →</td>
<td>$(3 + 5) + 2 = 3 + (5 + 2)$</td>
<td>$(6 \cdot 2) \cdot 7 = 6 \cdot (2 \cdot 7)$</td>
</tr>
<tr>
<td>parenthesis</td>
<td>$(x + y) + z = x + (y + z)$</td>
<td>$(x \cdot y) \cdot z = x \cdot (y \cdot z)$</td>
</tr>
<tr>
<td><strong>Distributive</strong></td>
<td>$a(b + c) = ab + ac$</td>
<td>$a(b − c) = ab − ac$</td>
</tr>
<tr>
<td>Hint: “Distribute” →</td>
<td>Multiplying a number by a group of numbers added together or subtracted</td>
<td></td>
</tr>
<tr>
<td>give out to</td>
<td>each other is the same as doing each multiplication separately.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$5(100 + 20) = 5 \cdot 100 + 5 \cdot 20$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$2(x + y) = 2x + 2y$</td>
<td></td>
</tr>
</tbody>
</table>
View the video lesson, take notes and complete the problems below.

<table>
<thead>
<tr>
<th>The commutative properties</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The associative properties</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Identity properties</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inverse properties</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distributive property</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-----------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>_____________________</td>
<td></td>
<td></td>
</tr>
<tr>
<td>_____________________</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **NOTE**: Subtraction and division do not have the associative and communicative properties.
EXERCISES

Identify the property that justifies each problem below.

1) \( w + 0 = w \)  
   Name: __________________________________________

2) \( -x + x = 0 \)  
   Name: __________________________________________

3) \( 3(u + v) = 3u + 3v \)  
   Name: __________________________________________

4) \( 5w(6z) = (6z)5w \)  
   Name: __________________________________________

5) \( (5)\frac{1}{5}x = 1x \)  
   Name: __________________________________________

6) \( 2 \cdot (4a)b = (2 \cdot 4)ab \)  
   Name: __________________________________________

7) \( (5x + 5) + 4 = 5x + (5 + 4) \)  
   Name: __________________________________________

8) \( 2x(3y + 7z) = 2x(7z + 3y) \)  
   Name: __________________________________________

Find the additive inverse and the multiplicative inverse for each problem below.

<table>
<thead>
<tr>
<th>Additive inverse</th>
<th>Multiplicative inverse</th>
</tr>
</thead>
<tbody>
<tr>
<td>9) ( 6 )</td>
<td></td>
</tr>
<tr>
<td>10) ( \frac{1}{2} )</td>
<td></td>
</tr>
<tr>
<td>11) (-2.5)</td>
<td></td>
</tr>
<tr>
<td>12) ( m ) (given ( m \neq 0 ))</td>
<td></td>
</tr>
</tbody>
</table>

Complete the following table by performing the indicated operations by computing the result in the parentheses first as the order of operations requires.

<table>
<thead>
<tr>
<th>Problem 1</th>
<th>Problem 2</th>
<th>Are the Results the Same?</th>
</tr>
</thead>
<tbody>
<tr>
<td>13) Addition</td>
<td>((5 + 7) + 3)</td>
<td>(5 + (7 + 3))</td>
</tr>
<tr>
<td></td>
<td>______________\</td>
<td>______________\</td>
</tr>
<tr>
<td>14) Subtraction</td>
<td>((10 - 5) - 4)</td>
<td>(10 - (5 - 4))</td>
</tr>
<tr>
<td></td>
<td>______________\</td>
<td>______________\</td>
</tr>
<tr>
<td>15) Multiplication</td>
<td>((2 \cdot 3) \cdot 4)</td>
<td>(2 \cdot (3 \cdot 4))</td>
</tr>
<tr>
<td></td>
<td>______________\</td>
<td>______________\</td>
</tr>
<tr>
<td>16) Division</td>
<td>((600 \div 30) \div 5)</td>
<td>(600 \div (30 \div 5))</td>
</tr>
<tr>
<td></td>
<td>______________\</td>
<td>______________\</td>
</tr>
</tbody>
</table>
Use the commutative or associative properties to perform the operations in the order you find most simple. Show all your work.

17) \((13 + 29) + 7\)  
18) \(5 \cdot (6 \cdot 8)\)

19) \((15 + 4) + 6\)  
20) \(4 \cdot 13 \cdot 5\)

Apply distributive property and combine like terms if possible.

21) \(8n(5 - m)\)  
22) \(7k(-6x + 6)\)

23) \(-6(1 + 6x)\)  
24) \(-8x(5 + 10n)\)

25) \(-(5 + 9a)\)  
26) \((n + 1)^2\)

27) \(4(x + 7) + 8(x + 4)\)  
28) \(-8(n + 6) - 8(n + 8)\)

29) \(-8x + 9(-9x + 9)\)  
30) \(4v - 7(1 - 8v)\)

31) \(-10(x - 2) - 3\)  
32) \(7(7 + 3v) + 10(3 - 10v)\)

33) \((x^2 - 8) - (2x^2 - 7)\)  
34) \(9(b + 10) + 5b\)

35) \(2n(-10n + 5) - 7(6 - 10n - 3m)\)

Log on to Canvas to take the section quiz
## CHAPTER REVIEW

### KEY TERMS AND CONCEPTS

Look for the following terms and concepts as you work through the workbook. In the space below, explain the meaning of each of these concepts and terms in your own words. Provide examples that are not identical to those in the text or in the media lesson.

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>A symbol that represents a number.</td>
</tr>
<tr>
<td>Algebraic expression</td>
<td>An expression that includes variables, numbers, and operations.</td>
</tr>
<tr>
<td>Coefficient</td>
<td>A number used as a factor in a multiplication.</td>
</tr>
<tr>
<td>Term</td>
<td>A part of an expression or equation.</td>
</tr>
<tr>
<td>Constant</td>
<td>A value that remains unchanged.</td>
</tr>
<tr>
<td>Like terms</td>
<td>Terms that have the same variable and can be combined.</td>
</tr>
<tr>
<td>Identity property of addition</td>
<td>Equality holds for any number. Usually 0 or 1.</td>
</tr>
<tr>
<td>Identity property of multiplication</td>
<td>Equality holds for any number. Usually 1 or 0.</td>
</tr>
<tr>
<td>Additive Inverse property</td>
<td>The opposite of a number.</td>
</tr>
<tr>
<td>Multiplicative Inverse property</td>
<td>The reciprocal of a number.</td>
</tr>
<tr>
<td>Associative property of addition</td>
<td>The grouping of numbers doesn't change the result.</td>
</tr>
<tr>
<td>Associative property of multiplication</td>
<td>The grouping of numbers doesn't change the result.</td>
</tr>
<tr>
<td>Commutative property of addition</td>
<td>The order of numbers doesn't change the result.</td>
</tr>
<tr>
<td>Commutative property of multiplication</td>
<td>The order of numbers doesn't change the result.</td>
</tr>
<tr>
<td>Distributive property</td>
<td>The distribution of multiplication across addition.</td>
</tr>
</tbody>
</table>
Tell the story of $x$ in each of the following expressions.

1) \( x + 2 - 2^1 \)  
2) \( 4(x^2 + 3) \)  
3) \( \frac{x+3}{5} \)

Write an algebraic expression that summarizes the stories below.

4) Step 1: Add 2 to $x$  
Step 2: Raise to the second power  
Step 3: Divide by 3

5) Step 1: Subtract 2 from $x$  
Step 2: Divide by 2  
Step 3: Add 1  
Step 4: Divide by 3

Evaluate the following expressions when \( a = -4 \), \( b = 2 \), and \( c = -1 \). Simplify your answers.

6) \( -c^3 \)  
7) \( \frac{6}{b} + c \)  
8) \( -a - b - c \)

9) \( \frac{b}{2} c - 3 \)  
10) \( a^2 + 2a - ac \)  
11) \( \frac{c}{2} + \frac{1}{2} b \)

12) Suppose sales tax in your city is 8.25%. Write an algebraic expression representing the sales tax for an item that cost $x$ dollars.

13) Will bought $c$ candy bars for $1.25 each. Write an algebraic expression for the total amount Will spent.

14) Consider the algebraic expression \( 2x + 3x^2 - \frac{x}{2} - x^{13} \)
   a) How many terms are there?___________
   b) Identify the constant terms.___________
   c) What is the coefficient of the first term?___________
   d) What is the coefficient of the second term?___________
   e) What is the coefficient of the third term?___________

Combine the like terms of the following expressions.

15) \( 2x - 3x - 4y \)  
16) \( 2x + y - 1 - y - 5x \)

17) \( 1x + 2x - x^2 + 3x - 2x^2 \)  
18) \( 4x - 2y + 3x + 4y + 2xy \)

Simplify. Combine like terms when possible.

19) \( (13 + 29) + 1 \)  
20) \( 4 \cdot (12 \cdot 2) \)  
21) \( n(5 - 2) \)

22) \( (n + 1)2 + n + 1 \)  
23) \( 9n - (n + 1) + 7n \)  
24) \( 2y + y(y + y) \)

25) \( z(z + 1) - z + 1 \)  
26) \( \frac{1}{2}(2x - 4) + 6 \)  
27) \( \frac{2x+4}{2} + 10(2x - \frac{1}{2}) \)
Find the additive and multiplicative inverse of the following problems.

28) 10

29) $\frac{3}{4}$

30) $\frac{-x}{1}$