CHAPTER 5: GRAPHING LINEAR EQUATIONS

Chapter Objectives
By the end of this chapter, students should be able to:
✓ Find the slope of a line from two points or a graph
✓ Find the equation of a line from its graph, the standard form, two given points
✓ Obtain equations of parallel and perpendicular lines

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SECTION 5.1 GRAPHING AND SLOPE

A. POINTS AND LINES

In this chapter, we will begin looking at the relationship between two variables. Typically one variable is considered to be the INPUT, and the other is called the OUTPUT. The input is the value that is considered first, and the output is the value that corresponds to or is matched with the input.

We write the input and its corresponding output as "(input, output)." This is known as an ordered pair.

For example,

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
<th>Ordered Pairs</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>-3</td>
<td>(4, -3)</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>(5, 8)</td>
</tr>
</tbody>
</table>

In an ordered pair, order matters. Let us take a look at the ordered pair (4,3). Since 4 appears first in this ordered pair, we know that 4 is the input. Likewise, since 3 appears second, we know that 3 is the output that belongs to 4. We can also refer to these numbers as coordinates.

To plot ordered pairs we use the Cartesian plane. The Cartesian plane is made up of a horizontal real number line (which we call the x-axis) and a vertical real number line (which we call the y-axis). The vertical and horizontal lines intersect at the point (0,0), which is called the origin. The Cartesian plane is divided into four quadrants.

To plot the ordered pair (4,3) we will look at the first coordinate, 4. We start at the origin and move to the right (the positive direction) by four units. Looking at the second coordinate, 3, we will then go up (in the positive direction) by three units. This is the point (4,3).
A line is made up of an infinite number of points. To draw a line, however, we only need two points. What a line represents are the solutions to a linear equation. An example of a linear equation is

\[ y = 2x + 1 \]

where \( x \) is the input, and \( y \) is the output. If we want to graph a linear equation, then we will need to make a table of inputs and outputs. Let us graph the linear equation above. For the table we are creating, we are allowed to pick any inputs we want. One person can pick the input 1 and another can pick the input 1,000. There is no wrong input you can choose for a linear equation, but we would like to keep things as simple as possible. Let us choose the following.

<table>
<thead>
<tr>
<th>Input (( x ) value)</th>
<th>Output (( y ) value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>?</td>
</tr>
<tr>
<td>1</td>
<td>?</td>
</tr>
<tr>
<td>(-2)</td>
<td>?</td>
</tr>
</tbody>
</table>

To find the corresponding outputs to the inputs we have chosen, we plug in one \( x \) value into the linear equation and solve for \( y \). Let us find all the outputs:

For \( x = 0 \):
\[ y = 2(0) + 1 \]
\[ y = 1 \]

For \( x = 1 \):
\[ y = 2(1) + 1 \]
\[ y = 2 + 1 \]
\[ y = 3 \]

For \( x = -2 \):
\[ y = 2(-2) + 1 \]
\[ y = -4 + 1 \]
\[ y = -3 \]

Filling in our chart

<table>
<thead>
<tr>
<th>Input (( x ) value)</th>
<th>Output (( y ) value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>(-2)</td>
<td>(-3)</td>
</tr>
</tbody>
</table>

Plotting these ordered pairs allows us to draw the line for the linear equation \( y = 2x + 1 \)
Two important points worth mentioning are the $x$ and $y$ intercepts of the line. The **$x$-intercept** of a line is the point $(x, 0)$, that is, the point where the line crosses the $x$-axis. The **$y$-intercept** of a line is the point $(0, y)$, that is, the point where the line crosses the $y$-axis. Below are some examples of $x$ and $y$ intercepts. The cross is indicated by an “$x$”.

![Graph showing $x$-intercept and $y$-intercept](image)

**MEDIA LESSON**

**Points and lines** (Duration 2:57)

*View the video lesson, take notes and complete the problems below*

- The positive numbers on the $x$-axis are located in what direction? _____________________
- The negative numbers on the $x$-axis are located in what direction? _____________________
- The positive numbers on the $y$-axis are located in what direction? _____________________
- The negative numbers on the $y$-axis are located in what direction? _____________________

We give _______________ to points on the $xy$-plane using these two number lines. First we give direction to the point going to _______________, then we give direction to the point going up.

**Example:** Graph the points. $(-2, 3), (4, -1), (-2, -4), (0, 3)$ and $(-1, 0)$

![Graph with points plotted](image)

**YOU TRY**
Plot and label the points.

a) \((-4, 2)\)

b) \((3, 8)\)

c) \((0, -5)\)

d) \((-6, -4)\)

e) \((5, 0)\)

f) \((2, -8)\)

g) \((0, 0)\)

B. Obtaining the Slope of a Line from Its Graph

The **slope** of a line is the measure of the line's steepness. We denote the slope of a line with the symbol \(m\). To find the slope of a line from its graph we look at the change in \(y\) over the change in \(x\), that is,

\[
m = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}}
\]

In order to determine the rise and run of a graph, let us look at an example. Let us graph the linear equation

\[y = x + 1\]

To find the rise we start at a well-defined point. In our graph above we started at \((-2, -1)\). Then locate a second well-defined point, in our case above we let that second point be \((2, 3)\). Now, starting at our initial point we rise up four units until we get to the exact same level as the second point. This is shown as a dotted vertical line above. Next, we move towards the second point which is four units to the right. This is shown as a dotted horizontal line above.

Since we rose up by four units, we say that the rise is 4.

Since we “ran” to the right by four units, we say that the run is 4.

Thus

\[m = \frac{\text{rise}}{\text{run}} = \frac{4}{4} = 1\]

So \(m = 1\).
NOTE: If the slope is **positive**, then the slope will be **rising** from left to right. If the slope is **negative**, then the slope will be **declining** from left to right.

We will now look at two special lines: the vertical line and the horizontal line.

A **vertical line** has the form \( x = c \), where \( c \) is a constant number. Here is an example of the vertical line \( x = 2 \).

If we were to pick the two well-defined points to be \((2, 4)\) and \((2, -4)\), then the rise would have a value of 8. However, the run will have a value of 0 since we do not move to the right or left.

Thus

\[
m = \frac{\text{rise}}{\text{run}} = \frac{8}{0} = \text{does not exist}
\]

Since we can’t divide by 0, the slope of the line does not exist.

A **horizontal line** has the form \( y = c \), where \( c \) is a constant number. Here is an example of the horizontal line \( y = 2 \).

If we were to pick the two well-defined points to be \((-3, 4)\) and \((3, 4)\), then the rise would have a value of 0 since we do not move up or down. The run, however, will have a value of 6.
Thus

\[ m = \frac{\text{rise}}{\text{run}} = \frac{0}{6} = 0 \]

Since 0 divided by anything is 0, our slope does exist and is 0.

To summarize:

- The slope of a vertical line does not exist
- The slope of a horizontal line does exist and has a value of 0.

**MEDIA LESSON**
Slope from two points. (Duration 5:00)

*View the video lesson, take notes and complete the problems below*

If we select _______________ points on a line we should be able to determine the _______________.

For example, if we are given the coordinates (3, 3) and (6, 5), we should be able to determine the _________________.

The slope of the two given coordinates is ________________, therefore the y-intercept is equal to______________.

We these two pieces of information, the linear equation is ________________.

**YOU TRY**

a) Find the slope of the line below.

b) Find the slope of the line below.
C. OBTAINING THE SLOPE OF A LINE FROM TWO POINTS

In the previous chapter we found the slope of a line by its graph. Another way to find the slope of a line (if we weren’t given its graph) is to look at any two points belonging to that line. Let us look at a modified definition of slope.

\[ m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1} \]

The last expression is what we are interested in. If we are given two points \((x_1, y_1)\) and \((x_2, y_2)\), then we just need to take the difference of the two \(y\) values and divide them by the difference of their respective \(x\) values. For example, if we have the points \((-1, 1)\) and \((1, 4)\), then

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 1}{1 - (-1)} = \frac{3}{2} \]

So \(m = \frac{3}{2}\).

View the video lesson, take notes and complete the problems below

Slope is calculated by ________________.

When we say rise over run we think of the rise as the change in ________________.

We think of the run as the change in ________________.

Follow the video and find the slope between \((-2, -5)\) and \((-17, 4)\).

YOU TRY

Find the slope between the given two points

a) \((-4, 3)\) and \((2, -9)\)  
b) \((-4, -1)\) and \((-4, 5)\)

c) \((4, 6)\) and \((2, -1)\)  
d) \((3, 1)\) and \((-2, 1)\)

EXERCISES

For problems 1-4 find the slope of the line.
Chapter 5

For problems 5-16 find the slope of the line through each ordered-pair.

5) \((-16, -14), (11, -14)\)  
6) \((-4, 14), (16, 8)\)  
7) \((12, -19), (6, 14)\)  

8) \((-5, 7), (18, 14)\)  
9) \((1, 2), (-6, -14)\)  
10) \((13, -2), (7, 7)\)  

11) \((-16, 2), (15, -10)\)  
12) \((8, 11), (-3, -13)\)  
13) \((11, -2), (1, 17)\)  

14) \((-2, 10), (-2, -15)\)  
15) \((-18, -5), (14, -3)\)  
16) \((19, 15), (5, 11)\)  

For problems 17-22 find the value of \(x\) or \(y\) so that the line through the points has the given slope.

17) \((2, 6) and (x, 2); m = \frac{-4}{7}\)  
18) \((-3, -2) and (x, 6); m = \frac{-8}{5}\)  

19) \((x, 5) and (8, 0); m = \frac{-5}{6}\)  
20) \((8, y) and (-2, 4); m = \frac{-1}{5}\)  

21) \((2, -5) and (3, y); m = 6\)  
22) \((6, 2) and (x, 6); m = \frac{-4}{5}\)
SECTION 5.2 EQUATIONS OF LINES

A. THE SLOPE-INTERCEPT FORMULA

The **slope-intercept form** of a linear equation is given by

\[ y = mx + b \]

Where \( m \) is the slope and \( b \) is the \( y \)-intercept (recall that the \( y \)-intercept is a **point**, so we really have \((0, b)\)).

When finding the equation of a line we would like the final result to be in slope-intercept form. This not only makes it easier to solve for \( y \) but it also gives us two important pieces of information: the **slope** and the **\( y \)-intercept** of the line.

**YOU TRY**

a) Find the equation of the line with slope 2 and \( y \)-intercept \((0, -3)\).

b) Find the equation of the line.

![Graph of a line]

**B. LINES IN SLOPE-INTERCEPT FORM**

Just by looking at a linear equation that is in slope-intercept form gives us the slope and \( y \)-intercept. With these two pieces of information we can readily graph its line. There will be times when the linear equation is not in slope-intercept form, and in those cases it could benefit us to manipulate the equation to where it is in slope-intercept form.

View the video lesson, take notes and complete the problems below

Give the equation of the line with a slope of \(-\frac{3}{4}\) and a \( y \)-intercept of 2.
We can put a linear equation in slope-intercept form to help us identify the ____________ and ____________. If the equation is not in this form, then we ______________ identify these key points of information. To put an equation in intercept form we _____________________.

Follow the video and give the slope and \( y \)-intercept of the graph

\[
y + 4 = \frac{2}{3}(x - 4)
\]

YOU TRY

a) Write the equation \( 3x - 9y = 6 \) in slope-intercept form. Find the slope and the \( y \)-intercept of the line.

C. GRAPHING LINES

Now that we know what information is given to us when a linear equation is in slope-intercept form and how to manipulate a linear equation to get it into slope-intercept form, let us use this form to start graphing lines.

View the video lesson, take notes and complete the problems below

We can graph an equation by identifying the ____________ and ____________. Once we have identified this key information we will start the graph at the ____________ and use the ____________ to change to the next point. Remember slope is ____________ over ____________.

Follow the video and put the linear equation \( 3x - 2y = 2 \) in slope intercept form. Then graph your answer.
YOU TRY

a) Graph \( y = \frac{1}{3}x - 2 \) by using the slope and \( y \)-intercept.

b) Graph the equation \( x + 2y = 10 \) using the slope and \( y \)-intercept.

D. VERTICAL AND HORIZONTAL LINES

Using the slope-intercept form when dealing with horizontal and vertical lines can help us understand them a little better.

Below is an example of a horizontal line that is in slope-intercept form

\[ y = 0x + b = b \]

Recall that a horizontal line has a slope of 0, hence the reason \( m = 0 \). If we let \( x = 1 \), then \( y = b \).

If we let \( x = -3 \), then \( y = b \). No matter what value we choose for \( x \), the \( y \) value will always be the same. **The equation for a horizontal line is therefore \( y = b \)** or its \( y \)-coordinate of the graph.

Unfortunately it is impossible for us to represent a vertical line in slope-intercept form because a vertical line’s slope does not exist. This tells us that there is no \( y \) in its equation. To represent a vertical line as an equation, we will simply make \( x \) equal to its \( x \)-coordinate of the graph. **The equation for a vertical line is therefore \( x = c \)**, where \( c \) is the intercept.
Chapter 5

MEDIA LESSON
Vertical and Horizontal (Duration 2:17)

View the video lesson, take notes and complete the problems below

Vertical lines, when graphed, will always go through the _______________. Vertical lines are always _______________ equals the _______________.

Horizontal lines, when graphed, will always go through the _______________. Horizontal lines are always _______________ equals the _______________.

1. Label the axes and graph \( y = -2 \)

2. Find the equation of the graph below.

YOU TRY

a) Graph \( y = 4 \).

b) Graph \( x = 4 \).

E. POINT-SLOPE FORMULA

The slope-intercept formula gives us two pieces of information that makes graphing relatively easy: the slope of the line and its \( y \)-intercept. Unfortunately we may not be given the \( y \)-intercept all the time. In these cases we can use the point-slope formula below
\[ y - y_1 = m(x - x_1) \]

where \( m \) is the slope and \((x_1, y_1)\) is a point on the line.

**YOU TRY**

a) Using the point-slope formula, write the equation of the line passing through the point \((1, 2)\) with a slope of \(\frac{1}{2}\). Write your final answer is slope-intercept form.

b) Using the point-slope formula, write the equation of the line passing through the point \((-2, 4)\) with a slope of \(-\frac{2}{5}\). Write your final answer is slope-intercept form.

**F. OBTAINING A LINE GIVEN TWO POINTS**

Let us look at the slope-intercept form again. In the previous section we used the point-slope formula to write an equation of a line given the slope and a point (not necessarily the \(y\)-intercept). Now we will look at another scenario: given only two points \((x_1, y_1)\) and \((x_2, y_2)\). If we are given two points (and no slope) then we can construct a linear equation in the following way:

1. We can use the two given points to find the slope using the slope formula.
2. Once we obtain the slope, we can then use the point-slope formula together with the slope and any of the two points that we were given to create the linear equation.
3. Once we have the equation we can then put it in slope-intercept form to help us graph.

One important fact is that to find the equation of a line we must have the_______________.

**MEDIA LESSON**

Point slope (Duration 5:00)

View the video lesson, take notes and complete the problems below

Give the equation of the line that passes through \((-3, 5)\) and has the slope of \(-\frac{2}{5}\).
Recall that the formula for slope is

\[ m = \]

Find the equation of the line through \((1, -4)\) and \((3, 5)\) and give the answer in slope-intercept form.

**YOU TRY**

a) Find the equation of the line passing through the points \((1, 2)\) and \((-1, -3)\) and write your final answer in slope-intercept form.

b) Find the equation of the line passing through the points \((3, -2)\) and \((1, 5)\) and write your final answer in slope-intercept form.
EXERCISES
For problems 1-3 write the equation of the line in slope-intercept form given the slope and the \( y \)-intercept.

1) \( m = \frac{1}{3} \), \( y \)-intercept = 1
2) \( m = -1 \), \( y \)-intercept = -2
3) \( m = \frac{2}{5} \), \( y \)-intercept = 5

For problems 4-6 write the equation of the line in slope-intercept form given the graph.

4)
5)
6)

For problems 7-15 write the equation of the line in slope-intercept form given the equation.

7) \( 2x + y = -1 \)
8) \( x = -8 \)
9) \( y - 4 = 4(x - 1) \)
10) \( y + 1 = -\frac{1}{2}(x - 4) \)
11) \( 6x - 11y = -70 \)
12) \( x - 7y = -42 \)
13) \( y - 3 = -\frac{2}{3}(x + 3) \)
14) \( 0 = x - 4 \)
15) \( x - 10y = 3 \)

For problems 16-21 sketch the graph of each line.

16) \( y = \frac{6}{5}x - 5 \)
17) \( x - y + 3 = 0 \)
18) \( -3y = -5x + 9 \)
19) \( y = -\frac{3}{2}x - 1 \)
20) \( 4x + 5 = 5y \)
21) \( -3y = 3 - \frac{3}{2}x \)

For problems 22-30 write the equation of the line in slope-intercept form given a point passing through the line and its slope.

22) \( (2, 2); m = \frac{1}{2} \)
23) \( (-4, 1); m = \frac{3}{4} \)
24) \( (0 - 5); m = -\frac{1}{4} \)
25) \( (-1, 4); m = -\frac{5}{4} \)
26) \( (2, 1); m = -\frac{1}{2} \)
27) \( (4, -3); m = -2 \)
28) \( (0, 2); m = -\frac{5}{4} \)
29) \( (2, 3); m = undefined \)
30) \( (2, 2); m = 0 \)

For problems 31-36 write the equation of the line in slope-intercept form given two points on the line.

31) \( (5, 1) \) and \( (-3, 0) \)
32) \( (3, 5) \) and \( (-5, 3) \)
33) \( (1, 3) \) and \( (-3, 3) \)
34) \( (-4, 1) \) and \( (4, 4) \)
35) \( (-5, 1) \) and \( (-1, -2) \)
36) \( (4, 1) \) and \( (1, 4) \)
SECTION 5.3 PARALLEL AND PERPENDICULAR LINES

A. THE SLOPE OF PARALLEL AND PERPENDICULAR LINES

<table>
<thead>
<tr>
<th>Parallel Lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lines $l_1$ and $l_2$ are said to be parallel to each other if their slopes are the same, that is, $m_1 = m_2$.</td>
</tr>
<tr>
<td>where $m_1$ is the slope of $l_1$ and $m_2$ is the slope of $l_2$.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Perpendicular Lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lines $l_1$ and $l_2$ are said to be perpendicular to each other if they have negative reciprocal slopes.</td>
</tr>
</tbody>
</table>

**MEDIA LESSON**
Slopes of parallel and perpendicular lines (Duration 5:23)

View the video lesson, take notes and complete the problems below

Parallel lines are two or more lines in a plane that never ________________.

Perpendicular lines are two or more lines that intersect at a ________________ angle.

Follow the video and find the slope of a line perpendicular to the line $y = -3x + 2$.

**YOU TRY**

a) Find the slope of a line parallel to $2y - 2x = 10$.

b) Find the slope of a line that is perpendicular to $2y - 2x = 10$. 
B. OBTAIN EQUATIONS FOR PARALLEL AND PERPENDICULAR LINES

Once we have the slope for a line that is perpendicular or parallel to another line, it is possible to find the equation for the perpendicular or parallel line given a point that is on one of these lines.

View the video lesson, take notes and complete the problems below

Parallel lines have the same _______________ and perpendicular lines have ________________ slopes. Once we know the slope and a point we can use the formula______________.

Follow the video and find the equation of the line parallel to the line $2x - 5y = 3$ that goes through the point $(5, 3)$.

YOU TRY

a) Find the equation of a line passing through $(1, 2)$ and parallel to $2x - 3y = 6$.

b) Find the equation of the line passing through $(6, -9)$ and perpendicular to $y = 2x + 1$. Write your final answer in slope-intercept form.
EXERCISES

For problems 1-3 find the slope of a line parallel to the given line.

1) \( y = 4x - 5 \)  
2) \( 7x + y = -2 \)  
3) \( y = -\frac{10}{3}x - 5 \)

For problems 4-6 find the slope of a line perpendicular to the given line.

4) \( x = 3 \)  
5) \( y = 4 \)  
6) \( 8x - 3y = -9 \)

For problems 7-16 find the equation of the line passing through the point and given the line to be parallel or perpendicular. Write your final answer in slope-intercept form.

7) \((5, 2); \text{ parallel to } y = \frac{7}{5}x + 4\)  
8) \((1, -2); \text{ perpendicular to } -x + 2y = 2\)

9) \((-1, 3); \text{ parallel to } y = -3x + 1\)  
10) \((1, 3); \text{ perpendicular to } -x + y = 1\)

11) \((1, 4); \text{ parallel to } y = \frac{7}{5}x + 2\)  
12) \((-3, -5); \text{ perpendicular to } 3x + 7y = 0\)

13) \((1, -1); \text{ parallel to } y = -3x - 1\)  
14) \((-2, -5); \text{ perpendicular to } y - 2x = 0\)

15) \((2, 5); \text{ parallel to } x = 0\)  
16) \((1, -1); \text{ perpendicular to } -x + y = 1\)
<table>
<thead>
<tr>
<th>KEY TERMS AND CONCEPTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Look for the following terms and concepts as you work through the workbook. In the space below, explain the meaning of each of these concepts and terms in your own words. Provide examples that are not identical to those in the text or in the media lesson.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th></th>
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<tbody>
<tr>
<td>Output</td>
<td></td>
</tr>
<tr>
<td>Cartesian plane</td>
<td></td>
</tr>
<tr>
<td>Origin</td>
<td></td>
</tr>
<tr>
<td>Ordered pair</td>
<td></td>
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<tr>
<td>$x$-intercept</td>
<td></td>
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<tr>
<td>$y$-intercept</td>
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<tr>
<td>Slope of a vertical line</td>
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<td>Slope of a horizontal line</td>
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<tr>
<td>Slope formula</td>
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<td>Slope-intercept form</td>
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<td>Point-slope form</td>
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<tr>
<td>Slope of parallel lines</td>
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<tr>
<td>Slope of perpendicular lines</td>
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