Find the sum of the series.[1]

$$1 - \ln(2) + \frac{(\ln(2))^2}{2!} - \frac{(\ln(2))^3}{3!} + \cdots$$

If we rewrite the series as

$$\frac{(\ln(2))^0}{0!} - \frac{(\ln(2))^1}{1!} + \frac{(\ln(2))^2}{2!} - \frac{(\ln(2))^3}{3!} + \cdots$$

we can recognize this as the power series

$$\frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

evaluated for $x = -\ln(2)$. Since

$$e^x = \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

we get

$$1 - \ln(2) + \frac{(\ln(2))^2}{2!} - \frac{(\ln(2))^3}{3!} + \cdots = e^{-\ln(2)} = \left(e^{\ln(2)}\right)^{-1} = 2^{-1}$$

Thus,

$$1 - \ln(2) + \frac{(\ln(2))^2}{2!} - \frac{(\ln(2))^3}{3!} + \cdots = \frac{1}{2}$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 772, #78.