Find the radius of convergence and the interval of convergence of the series.[1]

$$\sum_{n=1}^{\infty} n^n x^n$$

When determining radius and interval of convergence, the Ratio Test is usually the go-to test, however, with all the factors of our series being raised to the n-th power, we'll use the Root Test.

Here, $a_n = n^n x^n$, so we compute

$$\begin{aligned}\lim_{n\to\infty} \sqrt[n]{|a_n|} &= \lim_{n\to\infty} \sqrt[n]{|n^n x^n|} \\ &= \lim_{n\to\infty} n \cdot |x|\end{aligned}$$

This limit becomes ∞ for any value $x \neq 0$. If $x = 0$, then $\lim_{n\to\infty} n \cdot |0| = 0$. Thus the radius of convergence is $R = 0$ and the interval of convergence is just the number 0, which we can write as $I = \{0\}$.

[1]Stewart, *Calculus, Early Transcendentals*, p. 751, #8.