

Calculus II, Section 11.6, #36
Absolute Convergence and the Ratio and Root Tests

Use any test to determine whether the series is absolutely convergent, conditionally convergent, or divergent.¹

$$\sum_{n=1}^{\infty} \frac{\sin(n\pi/6)}{1+n\sqrt{n}}$$

In this series, note that the numerator is always less than or equal to 1, while the denominator behaves like $n^{3/2}$. Thus, we suspect convergence. We have

$$1+n\sqrt{n} > n\sqrt{n}$$
$$\frac{1}{1+n^{3/2}} < \frac{1}{n^{3/2}}$$

and since $\sin(n\pi/6) \leq 1$

$$\frac{\sin(n\pi/6)}{1+n^{3/2}} < \frac{1}{n^{3/2}}$$

So the series

$$\sum_{n=1}^{\infty} \frac{\sin(n\pi/6)}{1+n\sqrt{n}}$$

is convergent by the direct Comparison Test with known convergent p -series $\sum \frac{1}{n^{3/2}}$.

Also, since $|\sin(n\pi/6)| \leq 1$, the positive-termed series would also be convergent (using the same argument), and thus the series

$$\sum_{n=1}^{\infty} \frac{\sin(n\pi/6)}{1+n\sqrt{n}}$$

is absolutely convergent.

¹Stewart, *Calculus, Early Transcendentals*, p. 743, #36.