Determine whether the series is absolutely convergent or conditionally convergent.[1]

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3+1}$$

Our series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3+1}$ is clearly an alternating series. The corresponding positive-termed series is

$$\sum_{n=1}^{\infty} \frac{1}{n^3+1}$$

If we can show that the positive-termed series converges, then we know the series is absolutely convergent. (If the positive-termed series converges, then the alternating series also converges.)

Note that

$$n^3+1 > n^3$$

so

$$\frac{1}{n^3+1} < \frac{1}{n^3}$$

The terms of our positive-termed series are less than the terms of the known convergent p-series $\sum \frac{1}{n^3}$, $p = 3 > 1$, so by the direct Comparison Test

$$\sum_{n=1}^{\infty} \frac{1}{n^3+1}$$

is convergent.

Since the positive-termed series is convergent, we know

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3+1}$$

is absolutely convergent.

[1] Stewart, *Calculus, Early Transcendentals*, p. 742, #4.