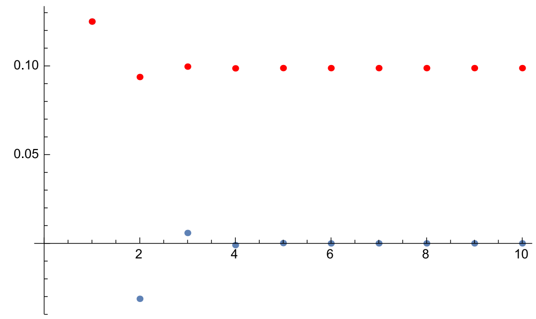
Graph both the sequence of terms and the sequence of partial sums on the same screen. Use the graph to make a rough estimate of the sum of the series. Then use the Alternating Series Estimation Theorem to estimate the sum correct to four decimal places.[1]

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{8^n}$$

On the coordinate system shown below, the terms of the sequence are in blue, and the terms of the sequence of partial sums are in red.



It seems that the sequence of partial sums is approaching 0.1, so $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{8^n} \approx 0.1$.

The series is alternating with $b_n = \frac{n}{8^n}$. The corresponding function is $f(x) = \frac{x}{8^x}$, so $f'(x) = \frac{8^x \cdot 1 - x \cdot 8^x \cdot \ln(8)}{(8^x)^2} = \frac{1 - x\ln(8)}{8^x} < 0$ for $x > \frac{1}{\ln(8)} \approx 0.5$. Thus $\left\{\frac{n}{8^n}\right\}$ is decreasing. Finally, $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{n}{8^n} \to \frac{\infty}{\infty}$. Applying l'Hospital's Rule, $\lim_{n\to\infty} \frac{n}{8^n} = \lim_{n\to\infty} \frac{1}{8^n \ln(8)} = 0$. So the sequence of b_n is positive, decreasing and has limit zero; thus we can apply the Alternating Series Estimation Theorem.

Note that

$$b_5 = \frac{5}{8^5} \approx 0.000152879$$

which is larger than the desired 0.0001.

$$b_6 = \frac{6}{8^6} \approx 0.00002289$$

which is smaller than the desired 0.0001, so we compute

$$\begin{aligned}\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{8^n} \approx s_5 &= \sum_{n=1}^{5} (-1)^{n-1} \frac{n}{8^n} \\ &= \frac{1}{8^1} - \frac{2}{8^2} + \frac{3}{8^3} - \frac{4}{8^4} + \frac{5}{8^5} \\ &\approx 0.0987854\end{aligned}$$

Adding b_6 to this approximation does not change the 4th decimal place, so the approximation of the sum of the series correct four decimal places, is

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{8^n} \approx 0.0988$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 736, #22.