

Determine whether the series is convergent or divergent. If it is convergent, find its sum.¹

$$\sum_{k=1}^{\infty} \frac{k^2}{k^2 - 2k + 5}$$

We suspect divergence since $\frac{k^2}{k^2}$ behaves like 1, so we'll apply the test for divergence.

$$\begin{aligned} \lim_{k \rightarrow \infty} \frac{k^2}{k^2 - 2k + 5} &= \lim_{k \rightarrow \infty} \frac{\frac{k^2}{k^2}}{\frac{k^2}{k^2} - \frac{2k}{k^2} + \frac{5}{k^2}} \\ &= \lim_{k \rightarrow \infty} \frac{1}{1 - \frac{2}{k} + \frac{5}{k^2}} \\ &= 1 \end{aligned}$$

Since $\lim_{k \rightarrow \infty} \frac{k^2}{k^2 - 2k + 5} = 1 \neq 0$, the series is divergent by the Test for Divergence.

¹Stewart, *Calculus, Early Transcendentals*, p. 716, #30.