

Calculus II, Section 11.2, #8
Series

Calculate the first eight terms of the sequence of partial sums correct to four decimal places. Does it appear that the series is convergent or divergent?¹

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n!}$$

Let's make a table of the partial sums.

| n | partial sum | s_n |
|-----|---|--------|
| 1 | $\frac{(-1)^{1-1}}{1!}$ | 1.0000 |
| 2 | $\frac{1}{1!} + \frac{(-1)^{2-1}}{2!}$ | 0.5000 |
| 3 | $\frac{1}{1!} - \frac{1}{2!} + \frac{(-1)^{3-1}}{3!}$ | 0.6667 |
| 4 | $\frac{1}{1!} - \frac{1}{2!} + \frac{1}{3!} + \frac{(-1)^{4-1}}{4!}$ | 0.6250 |
| 5 | $\frac{1}{1!} - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \frac{1}{5!}$ | 0.6333 |
| 6 | $\frac{1}{1!} - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \frac{1}{5!} - \frac{1}{6!}$ | 0.6319 |
| 7 | $\frac{1}{1!} - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \frac{1}{5!} - \frac{1}{6!} + \frac{1}{7!}$ | 0.6321 |
| 8 | $\frac{1}{1!} - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \frac{1}{5!} - \frac{1}{6!} + \frac{1}{7!} - \frac{1}{8!}$ | 0.6321 |

It appears that the series is convergent and $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n!} \approx 0.6321$.

¹Stewart, *Calculus, Early Transcendentals*, p. 715, #8.