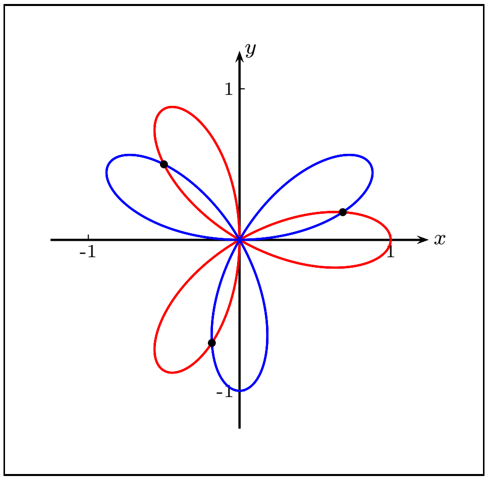
Calculus II, Section 10.4, #40
Areas and Lengths in Polar Coordinates

Find all points of intersection of the given curves.[1]

$r = \cos(3\theta), \quad r = \sin(3\theta)$

Here's the graph of the two curves.



Clearly, the pole is on both curves. Since we can represent the pole as (0,0) or $\left(0,\frac{\pi}{2}\right)$ (and an infinite number of other ways as well), the pole is a solution to both equations.

We solve

$$\begin{aligned} \sin(3\theta) &= \cos(3\theta) \\ \tan(3\theta) &= 1 \end{aligned}$$

From the unit circle, we know

$$\begin{aligned} 3\theta &= \frac{\pi}{4} + k\pi \\ \theta &= \frac{\pi}{12} + \frac{k\pi}{3} \\ &= \frac{\pi + 4k\pi}{12} \\ &= \frac{\pi}{12}(4k+1) \end{aligned}$$

When $k = 0$, $\theta = \frac{\pi}{12}$, and we get the polar point $\left(\cos\left(\frac{\pi}{4}\right), \frac{\pi}{12}\right) = \left(\frac{\sqrt{2}}{2}, \frac{\pi}{12}\right)$ or the rectangular point $\approx (0.6830, 0.1830)$.

When $k = 1$, $\theta = \frac{5\pi}{12}$, and we get the polar point $\left(\cos\left(\frac{5\pi}{4}\right), \frac{5\pi}{12}\right) = \left(-\frac{\sqrt{2}}{2}, \frac{5\pi}{12}\right)$ or the rectangular point $\approx (-0.1830, -0.6830)$.

When $k = 2$, $\theta = \frac{9\pi}{12}$, and we get the polar point $\left(\cos\left(\frac{9\pi}{4}\right), \frac{9\pi}{12}\right) = \left(\frac{\sqrt{2}}{2}, \frac{9\pi}{12}\right)$ or the rectangular point $(-0.5, 0.5)$.

[1]Stewart, *Calculus, Early Transcendentals*, p. 673, #40.