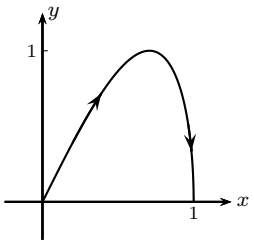
Set up an integral that represents the area of the surface obtained by rotating the given curve about the x-axis. Then use your calculator to find the surface area correct to four decimal places.[1]

$$x = \sin(t), \quad y = \sin(2t), \quad 0 \le t \le \pi/2$$



In terms of the parameter t, the surface area, S, is given by

$$\begin{aligned} S &= \int_{t=0}^{t=\pi/2} 2\pi y \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\, \mathrm{d}t \\ x &= \sin(t) \end{aligned}$$

so

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= \cos(t) \\ \left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 &= \cos^2(t) \end{aligned}$$

and

$$y = \sin(2t)$$

so

$$\begin{aligned} \frac{\mathrm{d}y}{\mathrm{d}t} &= 2\cos(2t) \\ \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 &= 4\cos^2(2t) \end{aligned}$$

Adding

$$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 = \cos^2(t) + 4\cos^2(2t)$$

Finally

$$S = \int_{t=0}^{t=\pi/2} 2\pi \sin(2t) \sqrt{\cos^2(t) + 4\cos^2(2t)}\, \mathrm{d}t$$

Using our TI-83, we get

$$\approx 8.0285$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 656, #58.