Find $\mathrm{d}x/\mathrm{d}y$ *and* $\mathrm{d}^2y/\mathrm{d}x^2$. *For which values of* t *is the curve concave upward?*[1]

$$x = t^2 + 1, \qquad y = e^t - 1$$

We know

$$x = t^2 + 1 \qquad\qquad y = \mathrm{e}^t - 1$$

so

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 2t \qquad\qquad \frac{\mathrm{d}y}{\mathrm{d}t} = \mathrm{e}^t$$

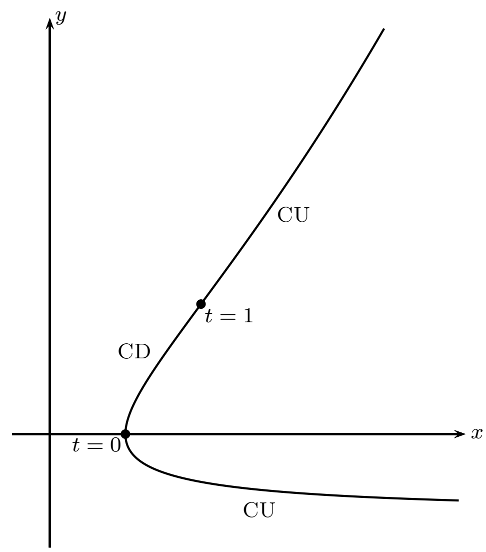
thus

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}} \\ &= \frac{\mathrm{e}^t}{2t}\end{aligned}$$

Now

$$\begin{aligned}\frac{\mathrm{d}^2y}{\mathrm{d}x} &= \frac{\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\mathrm{d}y}{\mathrm{d}x}\right]}{\frac{\mathrm{d}x}{\mathrm{d}t}} \\ &= \frac{\frac{2t\cdot \mathrm{e}^t - \mathrm{e}^t\cdot 2}{(2t)^2}}{2t} \\ &= \frac{2\mathrm{e}^t\,(t-1)}{8t^3} \\ &= \frac{\mathrm{e}^t\,(t-1)}{4t^3}\end{aligned}$$

The curve is concave up wherever $\frac{\mathrm{d}^2y}{\mathrm{d}x} > 0$. Since $\mathrm{e}^t > 0$ for all values of t, $\frac{\mathrm{d}^2y}{\mathrm{d}x} > 0$ when $t-1$ and t^3 have the same sign. If $t < 0$, $t^3 < 0$ and $t - 1 < 0$, so the curve is concave up. If $t > 0$, the denominator is positive, but the numerator is positive when $t > 1$. Thus the curve is concave up for $t < 0$ and $t > 1$.



[1]Stewart, *Calculus, Early Transcendentals*, p. 655, #14.