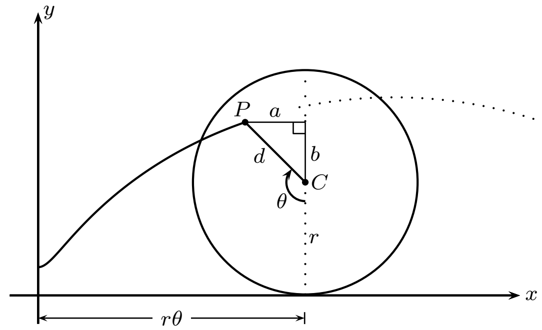
Let P be a point at a distance d from the center of a circle of radius s. The curve traced out by P as the circle rolls along a straight line is called a ***trochoid****. (Think of the motion of a point on a spoke of a bicycle wheel.) The cycloid is the special case of a trochoid with $d = r$. Using the same parameter θ as for the cycloid, and assuming the line is the x-axis and $\theta = 0$ when P is at one of its lowest points, show the the parametric equations of the trochiod are*[1]

$$x = r\theta - d\sin(\theta) \qquad y = r - d\cos(\theta)$$

We draw a diagram similar to the diagram we used for the cycloid.



From the right triangle with sides a, b, and d and some old trig. identities, we get

$$\begin{aligned} a &= d\sin(\pi - \theta) \\ &= d\sin(\theta) \end{aligned} \qquad \begin{aligned} b &= d\cos(\pi - \theta) \\ &= -d\cos(\theta) \end{aligned}$$

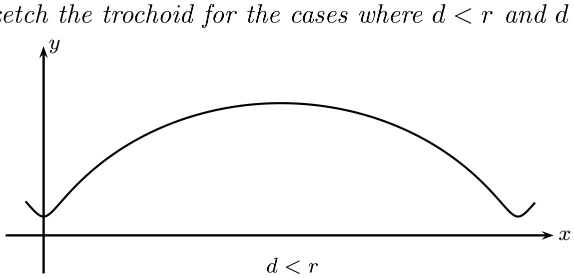
Since

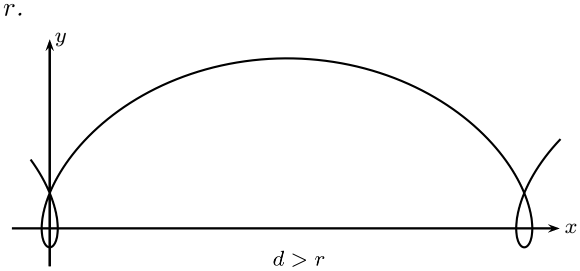
$$x = r\theta - a \qquad y = r + b$$

we get

$$x = r\theta - d\sin(\theta) \qquad y = r - d\cos(\theta)$$

Sketch the trochoid for the cases where $d < r$ and $d > r$.





[1] Stewart, *Calculus, Early Transcendentals*, p. 645, #40.