

Calculus II, Section 10.1, #18
Curves Defined by Parametric Equations

(a) Eliminate the parameter to find a Cartesian equation of the curve.¹

$$x = \tan^2(\theta), \quad y = \sec(\theta), \quad -\pi/2 < \theta < \pi/2$$

We know

$$1 + \tan^2(\theta) = \sec^2(\theta)$$

Substituting,

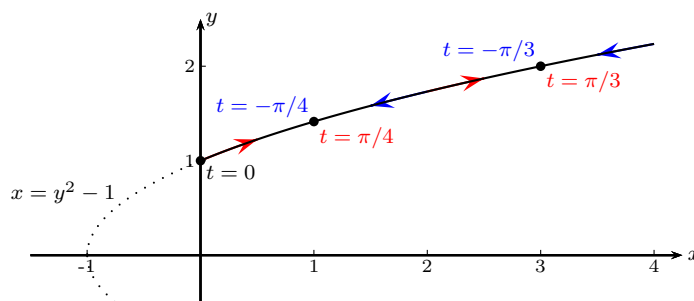
$$\begin{aligned} 1 + x &= y^2 \\ x &= y^2 - 1 \end{aligned}$$

If $-\pi/2 < \theta < \pi/2$, then $-\infty < \tan(\theta) < \infty$, so $-0 \leq \tan^2(\theta) < \infty$, and thus $0 \leq x$.

Also, if $-\pi/2 < \theta < \pi/2$, then $\sec(\theta) \geq 1$, so the graph of the parametric equations is in the first quadrant, with y -values always greater than or equal to one.

(b) Sketch the curve by and indicate with an arrow the direction in which the curve is traced as the parameter increases.

Here's the graph:



As t increases towards 0 from $-\pi/2$, the point moves to the left along the parabola towards $(0,1)$; this is shown in blue. At $t = 0$, the direction changes and as t continues to increase, the point moves to the right along the parabola; this is shown in red.

¹Stewart, *Calculus, Early Transcendentals*, p. 645, #18.