

Calculus II, Section 10.1, #8  
Curves Defined by Parametric Equations

---

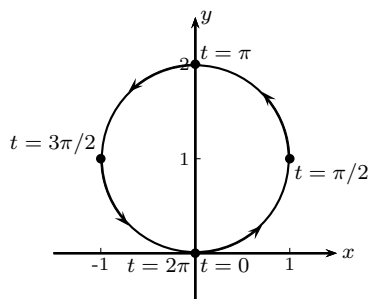
- (a) Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as  $t$  increases.<sup>1</sup>

$$x = \sin(t), \quad y = 1 - \cos(t), \quad 0 \leq t \leq 2\pi$$

Let's make a table of values with  $t$  as the independent variable, and  $x$  and  $y$  as functions of  $t$ .

$t$	0	$\pi/2$	$\pi$	$3\pi/2$	$2\pi$
$x$	0	1	0	-1	0
$y$	0	1	2	1	0

Here's the graph:



- (b) Eliminate the parameter to find a Cartesian equation of the curve.

We have

$$x = \sin(t) \qquad y = 1 - \cos(t)$$

so

$$\begin{aligned} x^2 &= \sin^2(t) & (y-1)^2 &= (-\cos(t))^2 \\ x^2 &= \sin^2(t) & (y-1)^2 &= \cos^2(t) \end{aligned}$$

Adding the two equations gives us

$$\begin{aligned} x^2 + (y-1)^2 &= \sin^2(t) + \cos^2(t) \\ x^2 + (y-1)^2 &= 1 \end{aligned}$$

Thus the graph of  $x = \sin(t)$ ,  $y = 1 - \cos(t)$ ,  $0 \leq t \leq 2\pi$ , is a circle of radius 1, centered at  $(0,1)$ .

---

<sup>1</sup>Stewart, *Calculus, Early Transcendentals*, p. 645, #8.