(a) *Assume that the carrying capacity for the US population is* 800 million. *Use it and the fact that the population was* 282 million *in* 2000 *to formulate a logistic model for the US population.*[1]

For a carrying capacity M and initial population P_0 at time $t = 0$, we have

$$P(t) = \frac{M}{1 + Ae^{-kt}} \qquad \text{where } A = \frac{M - P_0}{P_0}$$

If we let $t = 0$ correspond to the year 2000, then for this scenario, $M = 800$, and $A = \frac{800-282}{282} = \frac{518}{282} = \frac{259}{141}$.

Substituting the values for M and A, we get

$$P(t) = \frac{800}{1 + \frac{259}{141}e^{-kt}}$$

At this point, this is the best we can do. We need another point to determine the value of k.

(b) *Determine the value of k in your model by using the fact that the population in* 2010 *was* 309 million.

Substituting $t = 10$ and $P = 309$, we get

$$\begin{aligned}
309 &= \frac{800}{1 + \frac{259}{141}e^{-k\cdot 10}} \\
309\left(1 + \frac{259}{141}e^{-10k}\right) &= 800 \\
1 + \frac{259}{141}e^{-10k} &= \frac{800}{309} \\
\frac{259}{141}e^{-10k} &= \frac{800}{309} - 1 \\
e^{-10k} &= \frac{141}{259} \cdot \frac{491}{309} \\
-10k &= ln\left(\frac{69231}{80031}\right) \\
k &= \frac{ln\left(\frac{69231}{80031}\right)}{-10} \\
k &\approx 0.0145
\end{aligned}$$

Thus our logistic model is $P(t) = \dfrac{800}{1 + \frac{259}{141}e^{-0.0145t}}$.

(c) *Use your model to predict the US population in* 2100 *and* 2200.

$$P(t) = \frac{800}{1 + \frac{259}{141}e^{-0.0145t}}$$

so

$$\begin{aligned}
P(100) &= \frac{800}{1 + \frac{259}{141}e^{-0.0145\cdot 100}} \\
&\approx 559.1
\end{aligned}$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 618, #10.

and

$$\begin{aligned} P(200) &= \frac{800}{1 + \frac{259}{141}e^{-0.0145\cdot 200}} \\ &\approx 726.6 \end{aligned}$$

So in 2100, the US population will be about 559.1 million and in 2200, about 726.6 million.

(d) *Use your model to predict the year in which the US population will exceed* 500 million.

We solve

$$\begin{aligned} 500 &= \frac{800}{1 + \frac{259}{141}e^{-0.0145t}} \\ 500\left(1 + \frac{259}{141}e^{-0.0145t}\right) &= 800 \\ 1 + \frac{259}{141}e^{-0.0145t} &= \frac{800}{500} \\ \frac{259}{141}e^{-0.0145t} &= \frac{300}{500} \\ e^{-0.0145t} &= \frac{141}{259}\cdot\frac{300}{500} \\ -0.0145t &= \ln\left(\frac{423}{1295}\right) \\ t &= \frac{\ln\left(\frac{423}{1295}\right)}{-0.0145} \\ t &\approx 77.2 \end{aligned}$$

Thus, the US population will reach 500 million in 2077.