

Boxes are labeled as containing 500 g of cereal. The machine filling the boxes produces weights that are normally distributed with standard deviation 12 g.¹

- (a) If the target weight is 500 g, what is the probability that the machine produces a box with less than 480 g of cereal?

Let X = amount, in grams, of cereal in a box; then the random variable X is normally distributed with mean 500 and standard deviation 12. Thus the probability density function is

$$f(x) = \frac{1}{12\sqrt{2\pi}} e^{-(x-500)^2/(2 \cdot 12^2)}$$

We want

$$P(0 \leq X \leq 480) = \int_{x=0}^{x=480} \frac{1}{12\sqrt{2\pi}} e^{-(x-500)^2/(288)} dx$$

Using a graphing calculator or WolframAlpha², we get

$$P(0 \leq X \leq 480) \approx 0.0478$$

Thus there is about a 4.78% probability of a box of cereal containing less than 480 g of cereal.

- (b) Suppose a law states that no more than 5% of a manufacturer's cereal boxes can contain less than the stated weight of 500 g. At what target should the manufacturer set its filling machine?

We are asked to find the mean μ so that $P(0 \leq X \leq 500) = 0.05$.

We substitute different values of μ until we find that the probability is 0.05. We compute

$$P(0 \leq X \leq 500) = \int_{x=0}^{x=500} \frac{1}{12\sqrt{2\pi}} e^{-(x-\mu)^2/(288)} dx$$

and put the results in a table:

μ	$P(0 \leq X \leq 500)$	
500	0.500	P is too big, so μ is too small
510	0.202	P is too big
515	0.106	P is too big
517	0.078	P is too big
519	0.057	getting close
520	0.048	P is too small, so μ is too big
519.5	0.052	P is too big (but close)
519.75	0.0498...	P is too small (but close)
519.73	0.05007...	P is too big (but very close)
519.74	0.04998...	P is too small (but very close)

Thus, the manufacturer should set the machine to distribute 519.74 g of cereal to be certain of no more than 5% of the boxes containing less than 500 g of cereal.

¹Stewart, *Calculus, Early Transcendentals*, p. 580, #16.

² $\int \frac{1}{12\sqrt{2\pi}} e^{-(x-500)^2/(288)} dx$ is **not** an elementary integral; thus the only option is a numerical evaluation.