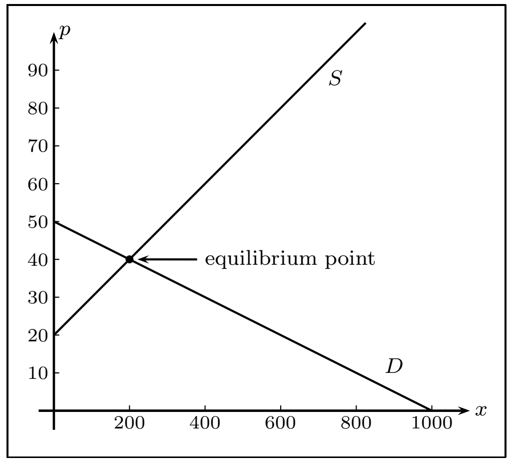
In a purely competitive market, the price of a good is naturally driven to the value where the quantity demanded by consumers matches the quantity made by producers, and the market is said to be in ***equilibrium****. These values are the coordinates of the point of intersection of the supply and demand curves.*[1]

(a) *Given the demand curve* $p = 50 - \frac{1}{20}x$ *and the supply curve* $p = 20 + \frac{1}{10}x$ *for a good, at what quantity and price is the market for the good in equilibrium?*

The demand and supply curves are sketched at right. To find the equilibrium point, we solve

$$\begin{aligned} 50 - \frac{1}{20}x &= 20 + \frac{1}{10}x \\ 30 &= \frac{1}{10}x + \frac{1}{20}x \\ 30 &= \frac{2x}{20} + \frac{x}{20} \\ 30 &= \frac{3x}{20} \\ 600 &= 3x \\ 200 &= x \end{aligned}$$



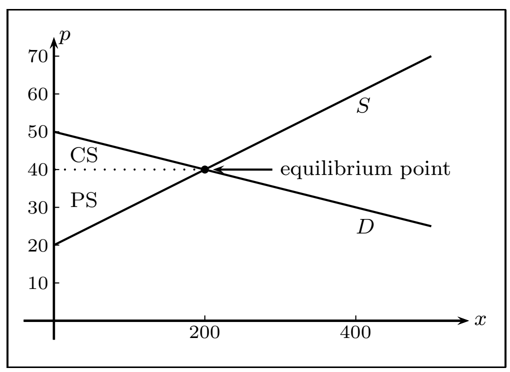
and if we substitute this value into either the demand or supply equation, we get

$$\begin{aligned} p &= 50 - \frac{1}{20} \cdot 200 \\ p &= 40 \end{aligned}$$

Thus the equilibrium point for the market is (200,40), i.e., if the price is \$40, we can expect a demand of 200 units.

(b) *Find the consumer surplus and the producer surplus when the market is in equilibrium. Illustrate by sketching the supply and demand curves and identifying the surpluses as areas.*

The regions representing the consumer surplus (CS) and producer surplus (PS) are labeled on the diagram at right and supply curves are sketched at right.



We have

$$\text{CS} = \int_{x=0}^{x=200} 50 - \frac{1}{20}x \, dx - 200 \cdot 40$$

and since the CS is a triangular region

$$\begin{aligned} &= \frac{1}{2} \cdot 200 \cdot 10 \\ &= 1000 \end{aligned}$$

Similarly

$$\text{PS} = 200 \cdot 40 - \int_{x=0}^{x=200} 20 + \frac{1}{10}x \, dx$$

and since the PS is a triangular region

$$\begin{aligned} &= \frac{1}{2} \cdot 200 \cdot 20 \\ &= 2000 \end{aligned}$$

[1]Stewart, *Calculus, Early Transcendentals*, p. 572, #8.