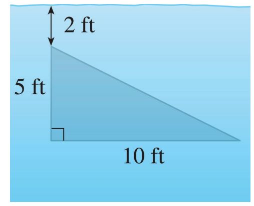
Calculus II, Section 8.3, #4
Applications to Physics and Engineering

The vertical plate is submerged (or partially submerged) in water and has the indicated shape. Explain how to approximate the hydrostatic force against one side of the plate by a Riemann sum. Then express the force as an integral and evaluate it.[1]

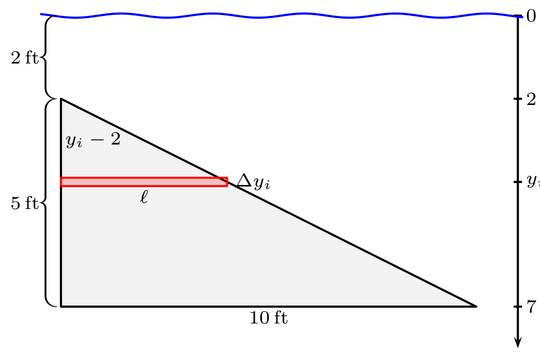


The plate in the diagram is measured using US Customary[2] units, so our computation of the hydrostatic force on the representative rectangle will be

$$F = \text{weight density} \cdot \text{area} \cdot \text{depth}$$

where weight density is measured in lb/ft^3, area is measured in ft^2, and, depth is measured in ft. Then we'll integrate to find the total hydrostatic force.

Let's determine the hydrostatic force on the representative rectangle.



The weight density of water is known[3], so we'll determine the area of the representative rectangle

$$\begin{aligned}\text{area of rep. rect.} &= \text{length of rep. rect.} \cdot \text{height of rep. rect.}\\ &= \ell \cdot \Delta y_i\end{aligned}$$

To find ℓ, we use similar triangles

$$\begin{aligned}\frac{y_i - 2}{\ell} &= \frac{5}{10}\\ \ell &= 2\left(y_i - 2\right)\end{aligned}$$

so

$$\text{area of rep. rect.} = 2\left(y_i - 2\right) \cdot \Delta y_i$$

The force on the representative rectangle is given by

$$\begin{aligned}\text{force on rep. rect.} &= \text{weight density} \cdot \text{area} \cdot \text{depth}\\ &= 62.5 \cdot 2\left(y_i - 2\right) \cdot \Delta y_i \cdot y_i\\ &= 125 y_i \left(y_i - 2\right) \Delta y_i\end{aligned}$$

We sum this expression for n sub-intervals to get the Riemann sum $\sum_{i=1}^{n} 125 y_i^* \left(y_i - 2\right) \Delta y_i^*$ where y_i^* is the sample point taken in the i-th subinterval. We won't usually write out the explicit Riemann sum, but this problem asked for it.

We create representative rectangles from $y = 2$ to $y = 7$, so the total hydrostatic force is given by

$$\begin{aligned}F &= \int_{y=2}^{y=7} 125y\left(y - 2\right)\, \mathrm{d}y\\ &= 125\left[\frac{y^3}{3} - y^2\right]_{y=2}^{y=7}\\ &= \frac{25000}{3}\\ &\approx 8333.3\,\text{lb}\end{aligned}$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 565, #4.
[2] Worst measuring system ever!
[3] Most sources give the weight density of water as $62.4\,\text{lb}/\text{ft}^3$. However, the Stewart textbook gives the weight density as $62.5\,\text{lb}/\text{ft}^3$. We will use $62.5\,\text{lb}/\text{ft}^3$ so our results will match those in the textbook, but be aware that in just about any other course, you'll use the more generally accepted value. Actually, you'll use the SI system, and won't have to use absurd units like feet and pounds.