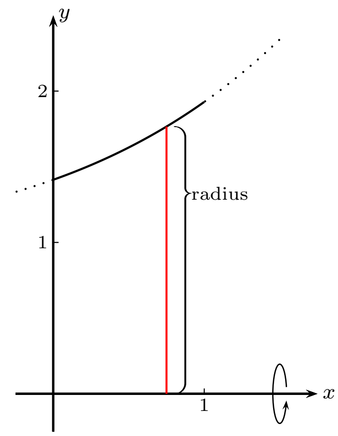
Find the exact area of the surface obtained by rotating the curve[1]

$$y = \sqrt{1 + \mathrm{e}^x}\,, \quad 0 \le x \le 1$$

about the x-axis.

The diagram at right shows the curve being revolved about the x-axis, along with a radius.



Since

$$y = \sqrt{1 + \mathrm{e}^x}$$

we have

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{1}{2\sqrt{1+\mathrm{e}^x}} \cdot \mathrm{e}^x \\ &= \frac{\mathrm{e}^x}{2\sqrt{1+\mathrm{e}^x}}\end{aligned}$$

In terms of x, the radius is $y = \sqrt{1+\mathrm{e}^x}$, the arc length is $\sqrt{1 + \left(\frac{\mathrm{e}^x}{2\sqrt{1+\mathrm{e}^x}}\right)^2}\,\mathrm{d}x$, and we integrate from $x = 0$ to $x = 1$. We get

$$\begin{aligned}S &= \int_{x=0}^{x=1} 2\pi\sqrt{1+\mathrm{e}^x} \cdot \sqrt{1 + \left(\frac{\mathrm{e}^x}{2\sqrt{1+\mathrm{e}^x}}\right)^2}\,\mathrm{d}x \\ &= \int_{x=0}^{x=1} 2\pi\sqrt{1+\mathrm{e}^x} \cdot \sqrt{1 + \frac{\mathrm{e}^{2x}}{4\left(1+\mathrm{e}^x\right)}}\,\mathrm{d}x \\ &= \int_{x=0}^{x=1} 2\pi\sqrt{1+\mathrm{e}^x} \cdot \sqrt{\frac{4\left(1+\mathrm{e}^x\right)}{4\left(1+\mathrm{e}^x\right)} + \frac{\mathrm{e}^{2x}}{4\left(1+\mathrm{e}^x\right)}}\,\mathrm{d}x \\ &= \int_{x=0}^{x=1} 2\pi\sqrt{1+\mathrm{e}^x} \cdot \frac{\sqrt{\mathrm{e}^{2x} + 4\mathrm{e}^x + 4}}{2\sqrt{1+\mathrm{e}^x}}\,\mathrm{d}x \\ &= \int_{x=0}^{x=1} \pi\sqrt{\left(\mathrm{e}^x + 2\right)^2}\,\mathrm{d}x \\ &= \pi\int_{x=0}^{x=1} \mathrm{e}^x + 2\,\mathrm{d}x \\ &= \pi\Big[\mathrm{e}^x + 2x\Big]_{x=0}^{x=1} \\ &= \pi\left[\left(\mathrm{e}^1 + 2\right) - \left(\mathrm{e}^0 + 2\cdot 0\right)\right] \\ &= \pi \cdot [\mathrm{e} + 1] \\ &= \pi\left(\mathrm{e}+1\right)\end{aligned}$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 555, #10.