(a) *If $g(x) = \frac{1}{\sqrt{x}-1}$, use your calculator or computer to make a table of approximate values of $\int_2^t g(x)\ \mathrm{d}x$ for $t = 5$, 10, 100, 1000, and 10,000. Does it appear that $\int_2^\infty g(x)\ \mathrm{d}x$ is convergent or divergent?*[1]

Using a TI-83, we get the following values.

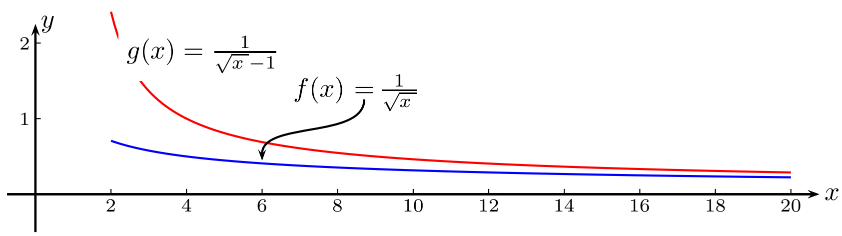
t	5	10	100	1000	10,000
$\int_2^t g(x)\ \mathrm{d}x$	3.830327	6.801200	23.328769	69.023361	208.124560

Judging from the progression of these values, it seems that $\int_2^\infty g(x)\ \mathrm{d}x$ is divergent.

(b) *Use the Comparison Theorem with $f(x) = \frac{1}{\sqrt{x}}$ to show that $\int_2^\infty g(x)\ \mathrm{d}x$ is divergent.*

For $x > 2$, we know $\sqrt{x} > \sqrt{x} - 1$, so $\frac{1}{\sqrt{x}} < \frac{1}{\sqrt{x}-1}$. Since $\int_2^\infty \frac{1}{\sqrt{x}}\ \mathrm{d}x$ is of the form $\int_2^\infty \frac{1}{x^p}\ \mathrm{d}x$ with $p = \frac{1}{2} \le 1$, it is divergent. Thus, by the Comparison Theorem, $\int_2^\infty \frac{1}{\sqrt{x}-1}\ \mathrm{d}x$ is divergent.

(c) Illustrate part (b) by graphing f and g on the same screen for $2 \le x \le 20$. Use your graph to explain intuitively why $\int_2^\infty g(x)\ \mathrm{d}x$ is divergent.



From the graph, it is clear that the values of the function $g(x) = \frac{1}{\sqrt{x}-1}$ are greater than the corresponding values of the function $f(x) = \frac{1}{\sqrt{x}}$, so $\int_2^t g(x)\ \mathrm{d}x$ is greater than the divergent $\int_2^t f(x)\ \mathrm{d}x$.

[1]Stewart, *Calculus, Early Transcendentals*, p. 535, #48.