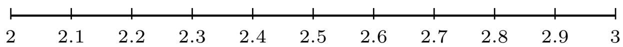
Calculus II, Section 7.7, #14
Approximate Integration

Use (a) the Trapezoidal Rule, (b) the Midpoint Rule, and (c) Simpson's Rule to approximate the given integral with the specified value of n. (Round your answers to six decimal places.)[1]

$$\int_2^3 \frac{1}{\ln(t)}\,\mathrm{d}t, \quad n = 10$$

For this integral[2] we are told to use $n = 10$, so our $\Delta t = \frac{3-2}{10} = \frac{1}{10}$. Here is our basic interval of integration:



Since we are given a formula for the integrand rather than data, we will be able to use these intervals for Midpoint Rule as well.

(a) For Trapezoidal Rule, we use

$$\int_a^b f(x)\,\mathrm{d}x \approx T_n = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$$

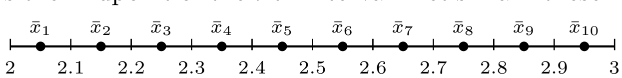
We have

$$\begin{aligned}
\int_2^3 \frac{1}{\ln(t)}\,\mathrm{d}t &\approx T_{10} \\
&= \frac{1/10}{2}\left[\frac{1}{\ln(2)} + 2\cdot\frac{1}{\ln(2.1)} + 2\cdot\frac{1}{\ln(2.2)} + 2\cdot\frac{1}{\ln(2.3)} + 2\cdot\frac{1}{\ln(2.4)}\right. \\
&\qquad \left. + 2\cdot\frac{1}{\ln(2.5)} + 2\cdot\frac{1}{\ln(2.6)} + 2\cdot\frac{1}{\ln(2.7)} + 2\cdot\frac{1}{\ln(2.8)} + 2\cdot\frac{1}{\ln(2.9)} + \frac{1}{\ln(3)}\right] \\
&= \frac{1}{20}\left[\frac{1}{\ln(2)} + \frac{2}{\ln(2.1)} + \frac{2}{\ln(2.2)} + \frac{2}{\ln(2.3)} + \frac{2}{\ln(2.4)}\right. \\
&\qquad \left. + \frac{2}{\ln(2.5)} + \frac{2}{\ln(2.6)} + \frac{2}{\ln(2.7)} + \frac{2}{\ln(2.8)} + \frac{2}{\ln(2.9)} + \frac{1}{\ln(3)}\right] \\
&\approx 1.119061
\end{aligned}$$

(b) For Midpoint Rule, we use

$$\int_a^b f(x)\,\mathrm{d}x \approx M_n = \Delta x\left[f(\bar{x}_1) + f(\bar{x}_2) + \cdots + f(\bar{x}_n)\right]$$

where $\bar{x}_i$ is the midpoint of the i-th interval. Let's mark these midpoints on the interval of integration:



We have

$$\begin{aligned}
\int_2^3 \frac{1}{\ln(t)}\,\mathrm{d}t &\approx M_{10} \\
&= \frac{1}{10}\left[\frac{1}{\ln(2.05)} + \frac{1}{\ln(2.15)} + \frac{1}{\ln(2.25)} + \cdots + \frac{1}{\ln(2.85)} + \frac{1}{\ln(2.95)}\right] \\
&\approx 1.118107
\end{aligned}$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 524, #14.

[2] The indefinite form of this integral, $\int \frac{1}{\ln(t)}\,\mathrm{d}t$ cannot be expressed in terms of elementary functions. This means an antiderivative that we could use with the FTC does not exist. Thus, the only way to compute the value of this definite integral is to use numerical approximation.

(c) For Simpson's Rule, we use

$$\int_a^b f(x)\ \mathrm{d}x \approx S_n = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$$

We have

$$\begin{aligned}
\int_2^3 \frac{1}{\ln(t)}\ \mathrm{d}t &\approx S_{10} \\
&= \frac{1/10}{3}\left[\frac{1}{\ln(2)} + 4\cdot\frac{1}{\ln(2.1)} + 2\cdot\frac{1}{\ln(2.2)} + 4\cdot\frac{1}{\ln(2.3)} + 2\cdot\frac{1}{\ln(2.4)}\right. \\
&\quad \left. +4\cdot\frac{1}{\ln(2.5)} + 2\cdot\frac{1}{\ln(2.6)} + 4\cdot\frac{1}{\ln(2.7)} + 2\cdot\frac{1}{\ln(2.8)} + 4\cdot\frac{1}{\ln(2.9)} + \frac{1}{\ln(3)}\right] \\
&= \frac{1}{30}\left[\frac{1}{\ln(2)} + \frac{4}{\ln(2.1)} + \frac{2}{\ln(2.2)} + \frac{4}{\ln(2.3)} + \frac{2}{\ln(2.4)}\right. \\
&\quad \left. + \frac{4}{\ln(2.5)} + \frac{2}{\ln(2.6)} + \frac{4}{\ln(2.7)} + \frac{2}{\ln(2.8)} + \frac{4}{\ln(2.9)} + \frac{1}{\ln(3)}\right] \\
&\approx 1.118428
\end{aligned}$$