Calculus II, Section 7.6, #46
Integration Using Tables and Computer Algebra Systems

Computer algebra systems sometimes need a helping hand from human beings. Try to evaluate

$$\int (1 + \ln(x)) \sqrt{1 + (x \ln(x))^2}\, dx$$

with a computer algebra system. If it doesn't return an answer, make a substitution that changes the integral into one that the CAS can integrate.[1]

We'll use Wolfram|Alpha (W|A).

Using Mathematica format, the input is

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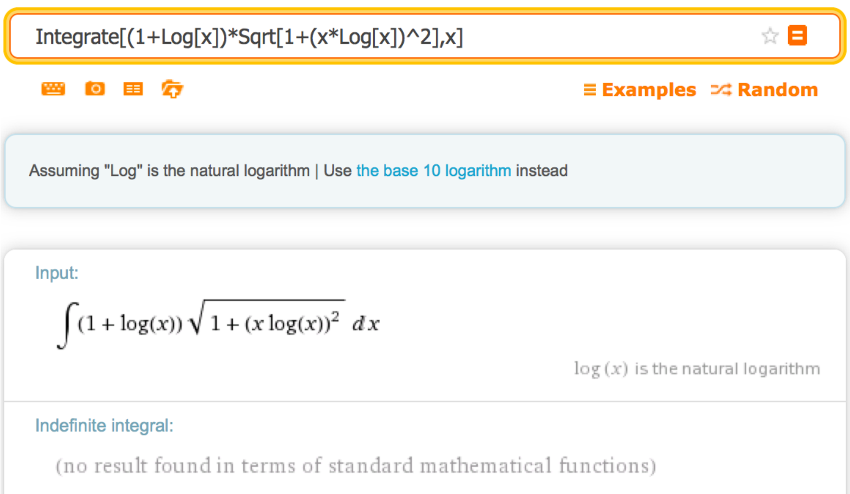
Integrate[(1+Log[x])*Sqrt[1+(x*Log[x])^2],x]
```

and we get



So W|A is unable to evaluate the integral.

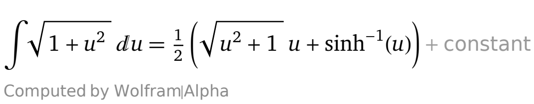
Let $u = 1 + (x\ln(x))^2$, then $du = 2(x\ln(x))\left(x \cdot \frac{1}{x} + \ln(x) \cdot 1\right) = 2(x\ln(x))(1 + \ln(x))\, dx$. This is not a good result for us—the factor $2(x\ln(x))$ is not present in the integrand—but this does show us that the derivative of $x\ln(x)$ *is* present in the integrand. Let's try again.

Let $u = x\ln(x)$, so $difu = \left(x \cdot \frac{1}{x} + \ln(x) \cdot 1\right) dx = (1 + \ln(x))\, dx$.

Our integral becomes

$$\int \sqrt{1 + u^2}\, du$$

and W|A gives us



Thus

$$\int (1 + \ln(x)) \sqrt{1 + (x\ln(x))^2}\, dx = \frac{1}{2}\left(x\ln(x)\sqrt{(x\ln(x))^2 + 1} + \sinh^{-1}(x\ln(x))\right) + C$$

[1]Stewart, *Calculus, Early Transcendentals*, p. 513, #46.