Calculus II, Section 7.4, #8
Integration of Rational Functions by Partial Fractions

Evaluate the integral.[1]

$$\int \frac{3t-2}{t+1}\,\mathrm{d}t$$

Consider the integral

$$\int \frac{3t-2}{t+1}\,\mathrm{d}t$$

Is the integrand one of our basic indefinite integrals? No. How about a basic u-substitution? No. Integration by parts? No. Powers of trig functions? No. Does the integrand include a trig. sub. radical? No. Is the integrand a rational function? YES!

The degree of the numerator is 1, and the degree of the denominator is 1. Since the degree of the numerator is greater than or equal to the degree of the denominator, we will do polynomial long division first.

$$t+1\overline{)\,3t-2} \quad\Rightarrow\quad \begin{array}{r} 3 \\ t+1\overline{)\,3t-2} \end{array} \quad\Rightarrow\quad \begin{array}{r} 3 \\ t+1\overline{)\,3t-2} \\ \underline{-(3t+3)} \end{array} \quad\Rightarrow\quad \begin{array}{r} 3 \\ t+1\overline{)\,3t-2} \\ \underline{-(3t+3)} \\ -5 \end{array}$$

So $\dfrac{3t-2}{t+1} = 3 + \dfrac{-5}{t+1} = 3 - \dfrac{5}{t+1}$ and we have

$$\begin{aligned} \int \frac{3t-2}{t+1}\,\mathrm{d}t &= \int 3 - \frac{5}{t+1}\,\mathrm{d}t \\ &= \int 3\,\mathrm{d}t - 5\int \frac{1}{t+1}\,\mathrm{d}t \\ &= 3t - 5\ln(t+1) + C \end{aligned}$$

Thus,

$$\int \frac{3t-2}{t+1}\,\mathrm{d}t = 3t - 5\ln(t+1) + C$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 501, #8.