Evaluate the integral.[1]

$$\int \tan^2(\theta)\sec^4(\theta)\ \mathrm{d}\theta$$

Tangents and secants ... what do we know?

$$1+\tan^2(\theta)=\sec^2(\theta) \qquad \sec^2(\theta)-1=\tan^2(\theta)$$

$$\frac{\mathrm{d}}{\mathrm{d}\theta}[\tan(\theta)]=\sec^2(\theta) \qquad \frac{\mathrm{d}}{\mathrm{d}\theta}[\sec(\theta)]=\sec(\theta)\tan(\theta)$$

Let's rewrite the original integral

$$\begin{aligned}
&\int \tan^2(\theta)\sec^4(\theta)\ \mathrm{d}\theta \\
&\quad=\int \tan^2(\theta)\sec^2(\theta)\sec^2(\theta)\ \mathrm{d}\theta \\
&\quad=\int \tan^2(\theta)\left(1+\tan^2(\theta)\right)\sec^2(\theta)\ \mathrm{d}\theta \\
&\quad=\int \left(\tan^2(\theta)+\tan^4(\theta)\right)\sec^2(\theta)\ \mathrm{d}\theta
\end{aligned}$$

and now we can do a basic u-substitution. Let $u=\tan(\theta)$, so $\mathrm{d}u=\sec^2(\theta)\ \mathrm{d}\theta$. We have

$$\begin{aligned}
&=\int u^2+u^4\,\mathrm{d}u \\
&=\frac{u^3}{3}+\frac{u^5}{5}+C
\end{aligned}$$

and now we substitute back to get the indefinite integral in terms of the original variable θ

$$=\frac{\tan^3(\theta)}{3}+\frac{\tan^5(\theta)}{5}+C$$

Thus,

$$\int \tan^2(\theta)\sec^4(\theta)\ \mathrm{d}\theta=\frac{\tan^3(\theta)}{3}+\frac{\tan^5(\theta)}{5}+C$$

[1]Stewart, *Calculus, Early Transcendentals*, p. 484, #22.