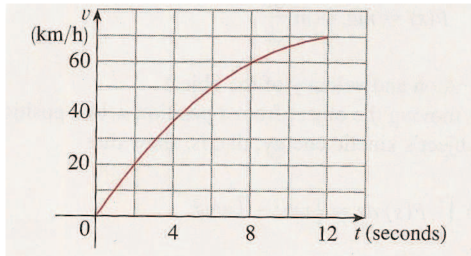
The velocity graph of an accelerating car is shown.[1]



(a) Use the Midpoint Rule to estimate the average velocity of the car during the first 12 seconds.

The average velocity is given by

$$v_{\text{ave}} = \frac{1}{12-0} \int_{t=0}^{t=12} v(t)\ \mathrm{d}t$$

We'll estimate the integral using the Midpoint Rule, and then complete the computation of the average velocity. Note that we will need to convert the rates from km/h to seconds.[2]

Since we must be able to evaluate the function at the midpoint of each interval, we will use the intervals [0,4] (so the midpoint is $t = 2$), [4,8], and [8,12]; for all of these, $\Delta t = 4$. When $t = 2$, $v(2) = 20\,\text{km/h} = \left(20\frac{\text{km}}{\text{h}}\right)\left(\frac{1\,\text{h}}{60\,\text{min}}\right)\left(\frac{1\,\text{min}}{60\,\text{sec}}\right) \approx 0.0056\frac{\text{km}}{\text{s}}$. When $t = 6$, $v(6) = 50 \approx 0.0139\frac{\text{km}}{\text{s}}$. Finally, when $t = 10$, $v(10) \approx 66 \approx 0.0183\frac{\text{km}}{\text{s}}$. From the Midpoint Rule, we get

$$\begin{aligned} \int_{t=0}^{t=12} v(t)\ \mathrm{d}t &\approx 4\,(0.0056 + 0.0139 + 0.0183) \\ &= 4 \cdot 0.0378 \\ &= 0.1512 \end{aligned}$$

So the average velocity is given by

$$\begin{aligned} v_{\text{ave}} &= \frac{1}{12-0} \int_{t=0}^{t=12} v(t)\ \mathrm{d}t \\ &\approx \frac{1}{12} \cdot 0.1512 \\ &\approx 0.0126 \end{aligned}$$

So the average velocity for the first twelve seconds is about $0.0126\frac{\text{km}}{\text{s}} = 45.36\frac{\text{km}}{\text{h}}$

(b) At what time was the instantaneous velocity equal to the average velocity?

From the graph, the velocity of $45.36\frac{\text{km}}{\text{h}}$ corresponds to a time of about 5.1 s.

[1]Stewart, *Calculus, Early Transcendentals*, p. 463, #16.
[2]Alternatively, we could convert from seconds to hours. We just must be consistent with our units.