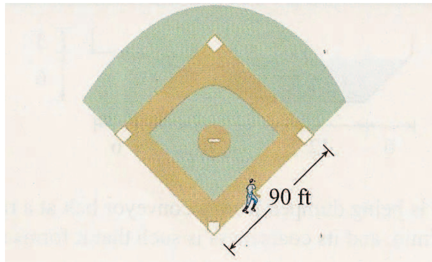
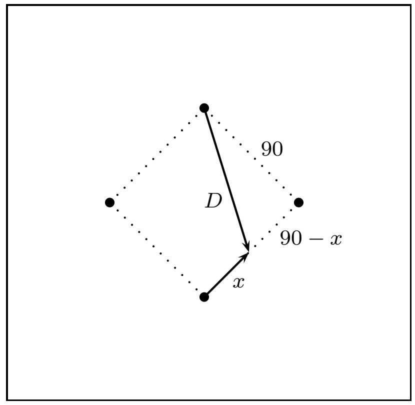
*A baseball diamond is a square with side* 90 ft. *A batter hits the ball and runs toward first base with a speed of* 24 ft/s.[1]




(a) *At what rate is his distance from second base decreasing when he is halfway to first base?*

If we let $x$ = distance batter has run at time $t$ and $D$ = distance from second base to the batter at time $t$, then we know $\frac{dx}{dt} = 24$ and we want $\frac{dD}{dt}$ when $x = 45$.




From Pythagorean Theorem

$$D^2 = (90 - x)^2 + 90^2$$

differentiating with respect to $t$

$$2D \cdot \frac{dD}{dt} = 2(90 - x)(-1)\frac{dx}{dt} + 0$$
$$D\frac{dD}{dt} = -(90 - x)\frac{dx}{dt}$$

Now, when $x = 45$,

$$D = \sqrt{(90 - 45)^2 + 90^2}$$
$$= \sqrt{(45)^2 + 90^2}$$

and substituting

$$\sqrt{(45)^2 + 90^2}\frac{dD}{dt} = -(90 - 45) \cdot 24$$
$$\frac{dD}{dt} = \frac{-45 \cdot 24}{\sqrt{(45)^2 + 90^2}}$$
$$\approx -10.73 \text{ ft/s}$$

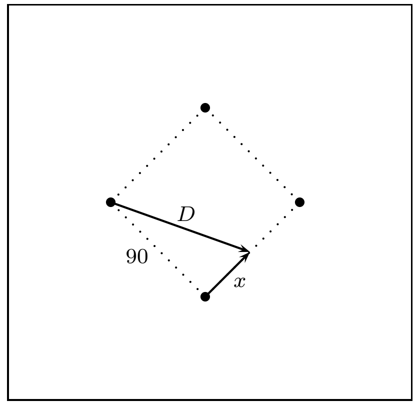
Thus, when the batter is halfway to first base, the distance between second base and the batter is decreasing at the rate of about 10.73 ft/s.

[1] Stewart, *Calculus, Early Transcendentals*, p. 249, #20.

*Continued* ⟹

(b) *At what rate is his distance from third base increasing at the same moment?*

If we let $x =$ distance batter has run at time $t$ and $D =$ distance from third base to the batter at time $t$, then we know $\frac{\mathrm{d}x}{\mathrm{d}t} = 24$ and we want $\frac{\mathrm{d}D}{\mathrm{d}t}$ when $x = 45$.




From Pythagorean Theorem

$$D^2 = x^2 + 90^2$$

differentiating with respect to $t$

$$2D \cdot \frac{\mathrm{d}D}{\mathrm{d}t} = 2x\frac{\mathrm{d}x}{\mathrm{d}t} + 0$$
$$D\frac{\mathrm{d}D}{\mathrm{d}t} = x\frac{\mathrm{d}x}{\mathrm{d}t}$$

Now, when $x = 45$,

$$D = \sqrt{45^2 + 90^2}$$

and substituting

$$\sqrt{45^2 + 90^2}\frac{\mathrm{d}D}{\mathrm{d}t} = 45 \cdot 24$$
$$\frac{\mathrm{d}D}{\mathrm{d}t} = \frac{45 \cdot 24}{\sqrt{45^2 + 90^2}}$$
$$\approx 10.73\,\mathrm{ft/s}$$

Thus, when the batter is halfway to first base, the distance between third base and the batter is increasing at the rate of about 10.73 ft/s.