Under certain circumstances a rumor spreads according to the equation

\[ p(t) = \frac{1}{1 + ae^{-kt}} \]

where \( p(t) \) is the proportion of the population that has heard the rumor at time \( t \) and \( a \) and \( k \) are positive constants.\(^1\)

(a) Find \( \lim_{t \to \infty} p(t) \)

\[
\lim_{t \to \infty} \frac{1}{1 + ae^{-kt}} = \lim_{t \to \infty} \frac{1}{1 + e^{kt}} = \frac{1}{1 + 0} \text{  Since } a \text{ is constant and } e^{kt} \to 0 \text{ as } t \to \infty \text{ because } k \text{ is positive.}
\]

(b) Find the rate of spread of the rumor.

We want the derivative.

\[
p'(t) = \frac{(1 + ae^{-kt}) \cdot 0 - 1 \cdot (0 + a \cdot e^{-kt} \cdot -k)}{(1 + ae^{-kt})^2} = \frac{ak e^{-kt}}{(1 + ae^{-kt})^2}
\]

(c) Graph \( p \) for the case \( a = 10, k = 0.5 \) with \( t \) measured in hours. Use the graph to estimate how long it will take 80% of the population to hear the rumor.

From the graph, it seems that 80% of the population will have heard the rumor in about 7.3 hrs.

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\(^1\)Stewart, Calculus, Early Transcendentals, p. 206, #84.