The letters r and θ represent polar coordinates. Write the equation using rectangular coordinates.[1]

$$r = \frac{3}{3 - \cos(\theta)}$$

We know $x = r\cos(\theta)$, so $\cos(\theta) = \frac{x}{r}$. Also, $x^2 + y^2 = r^2$ and, if $r > 0$, then $r = \sqrt{x^2 + y^2}$. With this information, let's substitute

$$\begin{aligned}
r &= \frac{3}{3 - \cos(\theta)} \\
\sqrt{x^2 + y^2} &= \frac{3}{3 - \frac{x}{r}} \\
\sqrt{x^2 + y^2} &= \frac{3}{3 \cdot \frac{r}{r} - \frac{x}{r}} \\
\sqrt{x^2 + y^2} &= \frac{3}{\frac{3r - x}{r}} \\
\sqrt{x^2 + y^2} &= \frac{3r}{3r - x} \\
\sqrt{x^2 + y^2} \cdot (3r - x) &= \frac{3r}{3r - x} \cdot (3r - x) \\
\sqrt{x^2 + y^2} \cdot (3r - x) &= 3r \\
3r - x &= \frac{3r}{\sqrt{x^2 + y^2}}
\end{aligned}$$

But $r = \sqrt{x^2 + y^2}$, so

$$\begin{aligned}
3r - x &= 3 \\
3r &= x + 3 \\
(3r)^2 &= (x + 3)^2 \\
9r^2 &= x^2 + 6x + 9 \\
9\left(x^2 + y^2\right) &= x^2 + 6x + 9 \\
8x^2 + 9y^2 - 6x - 9 &= 0
\end{aligned}$$

This isn't a bad equation, but let's complete the square in x to eliminate that $-6x$ term.

$$\begin{aligned}
8x^2 - 6x \quad + 9y^2 &= 9 \\
8\left(x^2 - \frac{6}{8}x \quad\right) + 9y^2 &= 9 \\
8\left(x^2 - \frac{3}{4}x \quad\right) + 9y^2 &= 9
\end{aligned}$$

[1] Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 568, #82.

Now $\frac{1}{2} \cdot \frac{-3}{4} = \frac{-3}{8}$ and $\left(\frac{-3}{8}\right)^2 = \frac{9}{64}$, so we add $\frac{9}{64}$ in the parentheses and $8 \cdot \frac{9}{64}$ to the right side

$$8\left(x^2 - \frac{3}{4}x + \frac{9}{64}\right) + 9y^2 = 9 + 8 \cdot \frac{9}{64}$$
$$8\left(x - \frac{3}{8}\right)^2 + 9y^2 = 9 + \frac{9}{8}$$
$$8\left(x - \frac{3}{8}\right)^2 + 9y^2 = \frac{81}{8}$$

or

$$64\left(x - \frac{3}{8}\right)^2 + 72y^2 = 81$$