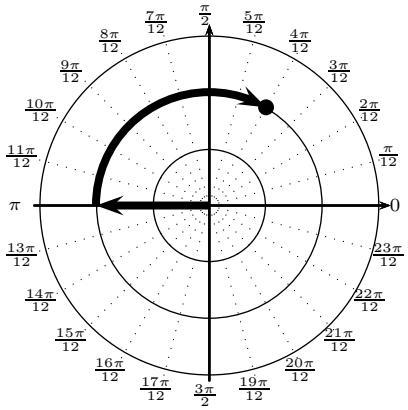
Precalculus, Section 9.1, #38
Polar Coordinates

Plot the point $\left(-2, -\frac{2\pi}{3}\right)$ *in polar coordinates, and find other polar coordinates* (r,θ) *of the point for which*[1]

(a) $r > 0, \quad -2\pi \le \theta < 0$

First we plot $\left(-2, -\frac{2\pi}{3}\right)$ in polar coordinates. We begin at the origin, and move -2 along the polar axis (x-axis). Then we rotate $-\frac{2\pi}{3}$.

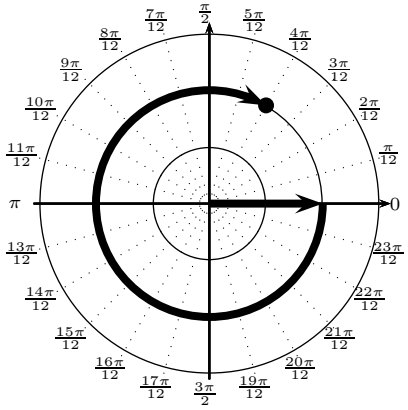
From the polar grid at right, we can see that the polar point $\left(-2, -\frac{2\pi}{3}\right)$ is equivalent to $\left(2, \frac{\pi}{3}\right)$.



Now we need to choose polar coordinate with $r > 0, \quad -2\pi \le \theta < 0$.

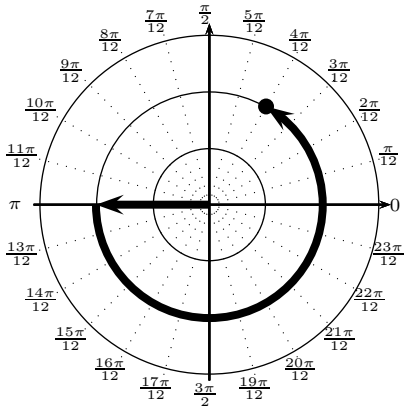
Since the radius is to be positive, we have $r = 2$. The rotation is to be negative, so we'll rotate $-\frac{20\pi}{12} = -\frac{5\pi}{3}$.

Thus the specified point is $\left(2, -\frac{5\pi}{3}\right)$.



(b) $r < 0, \quad 0 \le \theta < 2\pi$

Since the radius is to be negative, we have $r = -2$. The rotation is to be positive and less than 2π, so we have $\theta = \frac{16\pi}{12} = \frac{4\pi}{3}$.



[1] Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 567, #38.

(c) $r > 0, \quad 2\pi \le \theta < 4\pi$

Since the radius is to be positive, we have $r = 2$. The rotation is to be positive and greater than 2π but less than 4π. From part (a), we know that the point is equivalent to $\left(2, \frac{\pi}{3}\right)$, so we just add another complete rotation to this point. Thus

$$\left(2, \frac{\pi}{3} + 2\pi\right) = \left(2, \frac{\pi}{3} + \frac{6\pi}{3}\right) = \left(2, \frac{7\pi}{3}\right)$$