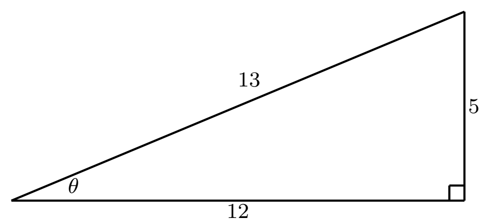
Find the exact value of the expression.[1]

$$\cos\left[\tan^{-1}\left(\frac{5}{12}\right) - \sin^{-1}\left(-\frac{3}{5}\right)\right]$$

The instructions ask for the exact value, so this is *not* a calculator exercise.

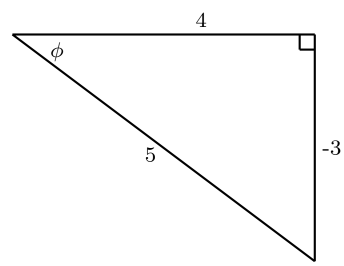
Recall that $\tan^{-1}\left(\frac{5}{12}\right)$ is the *angle* between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose tangent is $\frac{5}{12}$.

We draw a triangle with an angle θ such that $\tan(\theta) = \frac{5}{12}$. Using the Pythagorean theorem to find the hypotenuse gives us the diagram at right.



Similarly, $\sin^{-1}\left(-\frac{3}{5}\right)$ is the *angle* between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose sine is $-\frac{3}{5}$.

We draw a triangle with an angle ϕ such that $\sin(\phi) = -\frac{3}{5}$. Using the Pythagorean theorem to find the adjacent leg gives us the diagram at right.



Now that we have representations of the inverse trig functions, we can evaluate the given expression. Since this expression includes $\cos(\alpha - \beta)$, we'll probably need to recall

$$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

So

$$\cos\left[\tan^{-1}\left(\frac{5}{12}\right) - \sin^{-1}\left(-\frac{3}{5}\right)\right] = \cos\left(\tan^{-1}\left(\frac{5}{12}\right)\right)\cdot\cos\left(\sin^{-1}\left(-\frac{3}{5}\right)\right) + \sin\left(\tan^{-1}\left(\frac{5}{12}\right)\right)\cdot\sin\left(\sin^{-1}\left(-\frac{3}{5}\right)\right)$$

and from the triangles above,

$$= \frac{12}{13}\cdot\frac{4}{5} + \frac{5}{13}\cdot -\frac{3}{5}$$
$$= \frac{48}{65} - \frac{15}{65}$$
$$= \frac{33}{65}$$

[1] Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 486, #78.