Establish the identity[1]

$$\frac{\cos(\theta) + \sin(\theta) - \sin^3(\theta)}{\sin(\theta)} = \cot(\theta) + \cos^2(\theta)$$

For this identity, either side is a complicated expression, so we can choose which we'd like to work with. Let's start with the left side and try to transform it into the right.

$$\begin{aligned}
\frac{\cos(\theta) + \sin(\theta) - \sin^3(\theta)}{\sin(\theta)} &= \frac{\cos(\theta)}{\sin(\theta)} + \frac{\sin(\theta) - \sin^3(\theta)}{\sin(\theta)} \\
&= \cot(\theta) + \frac{\sin(\theta)\left(1 - \sin^2(\theta)\right)}{\sin(\theta)} \\
&= \cot(\theta) + \left(1 - \sin^2(\theta)\right) \\
&= \cot(\theta) + \cos^2(\theta)
\end{aligned}$$

Thus

$$\frac{\cos(\theta) + \sin(\theta) - \sin^3(\theta)}{\sin(\theta)} = \cot(\theta) + \cos^2(\theta)$$

On the other hand, we could start with the right side and try to transfom it into the left.

$$\begin{aligned}
\cot(\theta) + \cos^2(\theta) &= \frac{\cos(\theta)}{\sin(\theta)} + \cos^2(\theta) \\
&= \frac{\cos(\theta)}{\sin(\theta)} + \cos^2(\theta) \cdot \frac{\sin(\theta)}{\sin(\theta)} \\
&= \frac{\cos(\theta)}{\sin(\theta)} + \frac{\cos^2(\theta)\sin(\theta)}{\sin(\theta)} \\
&= \frac{\cos(\theta) + \cos^2(\theta)\sin(\theta)}{\sin(\theta)} \\
&= \frac{\cos(\theta) + \left(1 - \sin^2(\theta)\right)\sin(\theta)}{\sin(\theta)} \\
&= \frac{\cos(\theta) + \sin(\theta) - \sin^3(\theta)}{\sin(\theta)}
\end{aligned}$$

Thus

$$\cot(\theta) + \cos^2(\theta) = \frac{\cos(\theta) + \sin(\theta) - \sin^3(\theta)}{\sin(\theta)}$$

or, as the original identity stated

$$\frac{\cos(\theta) + \sin(\theta) - \sin^3(\theta)}{\sin(\theta)} = \cot(\theta) + \cos^2(\theta)$$

[1] Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 475, #84.