

Precalculus, Section 7.3, #80
Trigonometric Equations

Solve the equation on the interval $0 \leq \theta \leq 2\pi$.¹

$$\sec(\theta) = \tan(\theta) + \cot(\theta)$$

Since we're not terribly familiar with the secant or cosecant functions, let's rewrite the functions in terms of sine and cosine.

$$\begin{aligned}\sec(\theta) &= \tan(\theta) + \cot(\theta) \\ \frac{1}{\cos(\theta)} &= \frac{\sin(\theta)}{\cos(\theta)} + \frac{\cos(\theta)}{\sin(\theta)}\end{aligned}$$

multiply by the least common denominator $\cos(\theta) \sin(\theta)$

$$\begin{aligned}\cos(\theta) \sin(\theta) \cdot \frac{1}{\cos(\theta)} &= \cos(\theta) \sin(\theta) \cdot \frac{\sin(\theta)}{\cos(\theta)} + \cos(\theta) \sin(\theta) \cdot \frac{\cos(\theta)}{\sin(\theta)} && \cos(\theta) \neq 0, \sin(\theta) \neq 0 \\ \sin(\theta) &= \sin^2(\theta) + \cos^2(\theta) \\ \sin(\theta) &= 1\end{aligned}$$

Based on our knowledge of the unit circle, we find

$$\theta = \frac{\pi}{2}$$

but note that $\cos\left(\frac{\pi}{2}\right) = 0$, which makes the LCD zero and cannot be a solution.

Thus, the equation has no real solutions.

¹Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 465, #80.