

Precalculus, Section 7.3, #68
Trigonometric Equations

Solve the equation on the interval $0 \leq \theta \leq 2\pi$.¹

$$\cos(\theta) - \sin(-\theta) = 0$$

Although we might be tempted to start using some inverse trig functions, let's use some simple algebra to rewrite the equation with a single trig function.

$$\begin{aligned}\cos(\theta) - \sin(-\theta) &= 0 \\ -\sin(-\theta) &= -\cos(\theta) \\ \sin(-\theta) &= \cos(\theta)\end{aligned}$$

and since $\sin(-\theta) = -\sin(\theta)$,

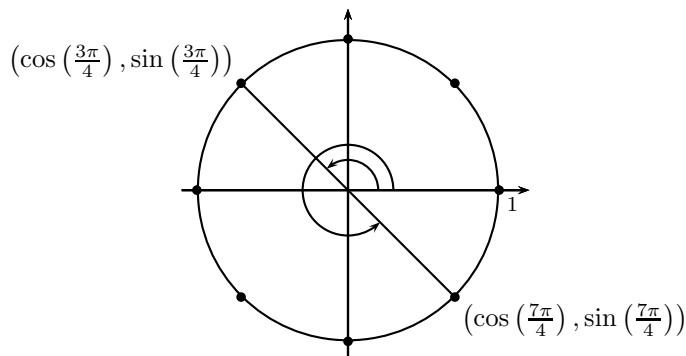
$$\begin{aligned}-\sin(\theta) &= \cos(\theta) \\ \frac{1}{\cos(\theta)} \cdot -\sin(\theta) &= \frac{1}{\cos(\theta)} \cdot \cos(\theta) \\ -\tan(\theta) &= 1 \\ \tan(\theta) &= -1\end{aligned}$$

So solving the original equation $\cos(\theta) - \sin(-\theta) = 0$ is equivalent to solving

$$\tan(\theta) = -1$$

which we can do with our knowledge of the unit circle. Since the value of the tangent function is negative, the angle θ must be in quadrant II or IV. Also, the angle must be where the sine and cosine functions have equal absolute values.

Thus the solutions are $\theta = \frac{3\pi}{4}$ and $\theta = \frac{7\pi}{4}$.



¹Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 465, #68.