

Precalculus, Section 6.6, #14  
Phase Shift; Sinusoidal Curve Fitting

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Find the amplitude, period, and phase shift of the function. Graph the function. Be sure to label key points and show a least two periods.<sup>1</sup>

$$y = -3 \cos\left(-2x + \frac{\pi}{2}\right)$$

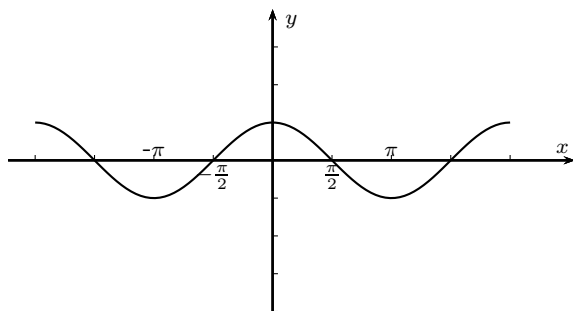
Let's factor  $-2$  inside the cosine function

$$y = -3 \cos\left(-2\left(x - \frac{\pi}{4}\right)\right)$$

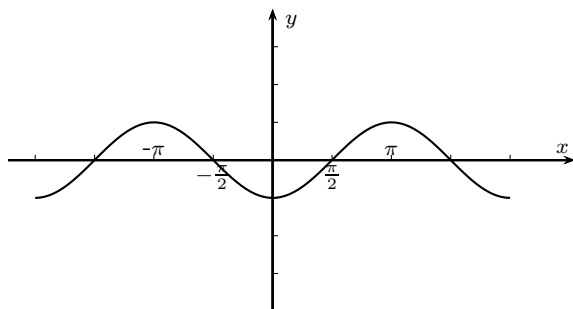
so we can use translations to graph this function.

We have a vertical flip and stretch by a factor of 3 from the  $-3$ , and a horizontal flip and stretch by a factor of  $\frac{1}{2}$  from the  $-2$ . Finally, there is a phase shift of  $\frac{\pi}{4}$  to the right.

Here are two cycles of  $y = \cos(x)$ :



The negative associated with the 2 generates a flip over the  $y$ -axis, however the parent is symmetrical about the  $y$ -axis, so there is no change. The negative associated with the 3 generates a flip over the  $x$ -axis. Here's the graph after these two flips are completed.  $y = -\cos(-x)$ :



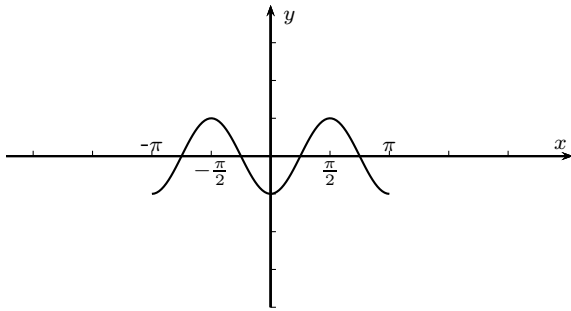
The coefficient 2 inside the cosine function generates a horizontal "stretch" by a factor of  $\frac{1}{2}$ , so all the  $x$ -values get multiplied by  $\frac{1}{2}$  (Some instructors refer to this as a dilation or compression.) Here's the graph after the horizontal stretch is completed.  $y = -\cos(-2x)$ :

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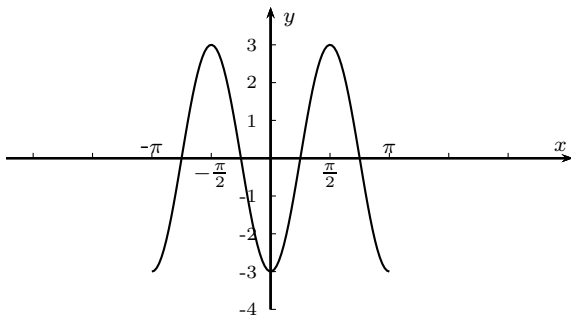
<sup>1</sup>Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 429, #14.

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The coefficient 3 outside the cosine function generates a vertical stretch by a factor of 3, so all the  $y$ -values get multiplied by 3. Here's the graph after the vertical stretch is completed.  $y = -3 \cos(-2x)$ :



Finally, there is a horizontal translation of  $\frac{\pi}{4}$  from the  $(x - \frac{\pi}{4})$  inside the cosine function. Here's the graph after the horizontal translation is completed.  $y = -3 \cos(-2(x - \frac{\pi}{4}))$ :

