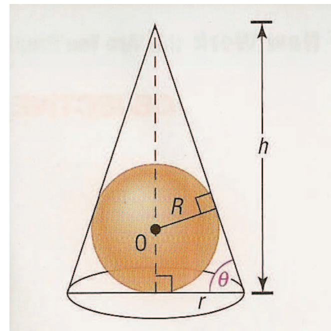
Designing Fine Decorative Pieces *A designer of decorative art plans to market solid gold spheres encased in clear crystal cones. Each sphere is of fixed radius R and will be enclosed in a cone of height h and radius r. See the illustration. Many cones can be used to enclose the sphere, each having a different slant angle θ. The volume V of the cone can be expressed as a function of the slant angle θ of the cone as*

$$V(\theta) = \frac{1}{3}\pi R^3 \frac{(1+\sec(\theta))^3}{(\tan(\theta))^2}, \qquad 0^\circ < \theta < 90^\circ$$



What volume V is required to enclose a sphere of radius 2 centimeters in a cone whose slant angle θ is 30°? 45°? 60°?[1]

Note that the volume is a function of θ and the fixed value R, so despite the long preamble and the complicated looking illustration, this is just a substitution problem followed by a calculator exercise.

Here, $R = 2$, so the function becomes

$$V(\theta) = \frac{1}{3}\pi R^3 \frac{(1+\sec(\theta))^3}{(\tan(\theta))^2}$$
$$V(\theta) = \frac{1}{3}\pi \cdot 2^3 \cdot \frac{(1+\sec(\theta))^3}{(\tan(\theta))^2}$$
$$V(\theta) = \frac{8}{3}\pi \frac{(1+\sec(\theta))^3}{(\tan(\theta))^2}$$

Now we substitute each given value.

$$V(30) = \frac{8}{3}\pi \frac{(1+\sec(30))^3}{(\tan(30))^2}$$
$$V(30) \approx 251.42\,\text{cm}^3$$

$$V(45) = \frac{8}{3}\pi \frac{(1+\sec(45))^3}{(\tan(45))^2}$$
$$V(45) \approx 117.88\,\text{cm}^3$$

$$V(60) = \frac{8}{3}\pi \frac{(1+\sec(60))^3}{(\tan(60))^2}$$
$$V(60) \approx 75.4\,\text{cm}^3$$

[1] Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 383, #128.