Population of an Endangered Species *Often environmentalists capture an endangered species and transport the species to a controlled environment where the species can produce offspring and regenerate its population. Suppose that six American bald eagles are captured, transported to Montana, and set free. Based on experience, the environmentalists expect the population to grow according to the model*

$$P(t) = \frac{500}{1 + 82.33\mathrm{e}^{-0.162t}}$$

where t is measured in years.[1]

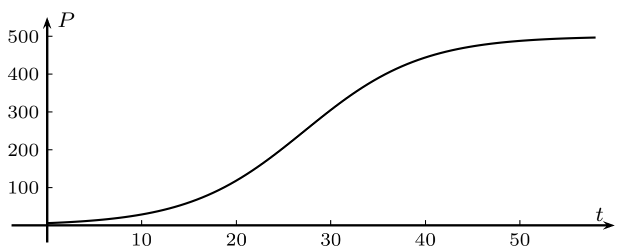
(a) *Determine the carrying capacity of the environment.*

The carrying capacity is the numerator of the logistic function, so $C = 500$ eagles.

(b) *What is the growth rate of the bald eagle?*

The growth rate is the absolute value of k; thus the growth rate is 16.2%.

(c) *Use a graphing utility to graph $P = P(t)$.*



(d) *What is the population after 3 years?*

Substituting $t = 3$ into the model gives

$$P(3) = \frac{500}{1 + 82.33\mathrm{e}^{-0.162\cdot 3}}$$
$$P(3) \approx 10\,\text{eagles}$$

(e) *When will the population be 300 eagles?*

We solve

$$\begin{aligned}
300 &= \frac{500}{1 + 82.33\mathrm{e}^{-0.162t}} \\
300\left(1 + 82.33\mathrm{e}^{-0.162t}\right) &= 500 \\
300 + 24699\mathrm{e}^{-0.162t} &= 500 \\
24699\mathrm{e}^{-0.162t} &= 200 \\
\mathrm{e}^{-0.162t} &= \frac{200}{24699} \\
\ln\left(\mathrm{e}^{-0.162t}\right) &= \ln\left(\frac{200}{24699}\right) \\
-0.162t &= \ln(200/24699) \\
t &= \frac{\ln(200/24699)}{-0.162}
\end{aligned}$$

so

$$t \approx 29.7\,\text{years}$$

[1] Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 337, #24.

(f) *How long does it take for the population to reach one-half of the carrying capacity?*

Since the carrying capacity is 500 eagles, we want to know when the population will reach 250 eagles. We solve

$$\begin{aligned}
250 &= \frac{500}{1+82.33\mathrm{e}^{-0.162t}} \\
250\left(1+82.33\mathrm{e}^{-0.162t}\right) &= 500 \\
250+20582.5\mathrm{e}^{-0.162t} &= 500 \\
20582.5\mathrm{e}^{-0.162t} &= 250 \\
\mathrm{e}^{-0.162t} &= \frac{250}{20582.5} \\
\ln\left(\mathrm{e}^{-0.162t}\right) &= \ln\left(\frac{250}{20582.5}\right) \\
-0.162t &= \ln(250/20582.5) \\
t &= \frac{\ln(250/20582.5)}{-0.162}
\end{aligned}$$

so

$$t \approx 27.2\,\text{years}$$