Solve the exponential equation. Express irrational solutions in exact form and as a decimal rounded to three decimal places. Verify your results using a graphing utility[1]

$$9^x - 3^{x+1} + 1 = 0$$

Since $3^2 = 9$ and $3^x \cdot 3^1 = 3^{x+1}$, we can write

$$\left(3^2\right)^x - 3^x \cdot 3^1 + 1 = 0$$

and since $\left(3^2\right)^x = 3^{2x} = 3^{x \cdot 2} = (3^x)^2$

$$(3^x)^2 - 3 \cdot 3^x + 1 = 0$$

Now we let $u = 3^x$ and substitute to get

$$u^2 - 3u + 1 = 0$$

Unfortunately, the left side does not factor, so we use the quadratic formula to get

$$\begin{aligned} u &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} \\ &= \frac{3 \pm \sqrt{5}}{2} \end{aligned}$$

So

$$u = \frac{3 + \sqrt{5}}{2} \quad \text{or} \quad u = \frac{3 - \sqrt{5}}{2}$$

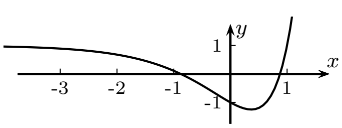
Now, we know $u = 3^x$, so

$$3^x = \frac{3 + \sqrt{5}}{2} \quad \text{or} \quad 3^x = \frac{3 - \sqrt{5}}{2}$$

and from the definition of logarithm,

$$\begin{aligned} &x = \log_3\left(\frac{3 + \sqrt{5}}{2}\right) \quad \text{or} \quad x = \log_3\left(\frac{3 - \sqrt{5}}{2}\right) \\ &x \approx 0.876 \quad \text{or} \quad x \approx -0.876 \end{aligned}$$

Since our original equation is equal to zero, solving with the TI-84 graphing calculator is equivalent to finding the zeros of the function. The graph is shown below.



Using the `calc:zero` function on the TI-84, the zeros of the function and thus solutions to the equation are $x \approx 0.876$ or $x \approx -0.876$.

[1]Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 315, #54.