

Use the given function f to:¹

$$f(x) = -2 \ln(x + 1)$$

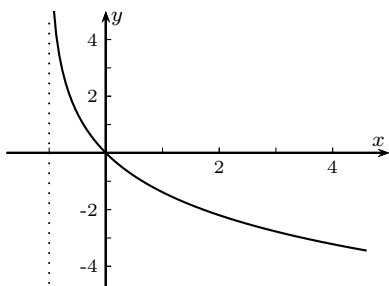
- (a) Find the domain of f .

Since the input to any logarithmic function must be positive, we need

$$\begin{aligned} x + 1 &> 0 \\ x &> -1 \end{aligned}$$

so in set builder notation, we write $\{x|x > -1\}$. Using interval notation, we write $(-1, \infty)$.

- (b) Graph f .



- (c) From the graph, determine the range and any asymptotes of f .

The range of f is all real numbers, *i.e.* $(-\infty, \infty)$. There is a vertical asymptote at $x = -1$.

- (d) Find f^{-1} , the inverse of f .

We write our function as

$$y = -2 \ln(x + 1)$$

and solve for x :

$$\begin{aligned} -\frac{1}{2}y &= -\frac{1}{2} \cdot -2 \ln(x + 1) \\ -\frac{1}{2}y &= \ln(x + 1) \end{aligned}$$

From the definition of logarithm (or "exponentiating both sides")

$$\begin{aligned} e^{-\frac{1}{2}y} &= e^{\ln(x+1)} \\ e^{-\frac{1}{2}y} &= x + 1 \\ e^{-\frac{1}{2}y} - 1 &= x \end{aligned}$$

We switch the variables and write the inverse of f :

$$f^{-1}(x) = e^{-\frac{1}{2}x} - 1$$

- (e) Find the domain and range of f^{-1} .

Since the range of f is $(-\infty, \infty)$, the domain of f^{-1} is $(-\infty, \infty)$. Since the domain of f is $(-1, \infty)$, the range of f^{-1} is $(-1, \infty)$.

¹Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 298, #76.

Precalculus
Logarithmic Functions

(f) Graph f^{-1} .

