Cost of a Commodity *The price p, in dollars, of a certain commodity and the quantity x sold obey the demand equation*

$$p = -\frac{1}{5}x + 200 \qquad 0 \le x \le 1000$$

Suppose that the cost C, in dollars,of producing x units is

$$C = \frac{\sqrt{x}}{10} + 400$$

Assuming that all items produced are sold, find the cost C as a function of the price p.[1]

We want $C(p)$.

We are given $C(x)$ and p as a function of x. We will do the necessary algebra to write x as a function of p, and then substitute that new function into C.

Let's solve for x as a function of p:

$$\begin{aligned} p &= -\frac{1}{5}x + 200 \\ p - 200 &= -\frac{1}{5}x \\ -5 \cdot (p - 200) &= -5 \cdot -\frac{1}{5}x \\ -5p + 1000 &= x \end{aligned}$$

Note that this is a linear function, so when $x = 0$, $p = 200$ and when $x = 1000$, $p = 0$.

Now let's substitute this expression for x into $C(x)$:

$$\begin{aligned} C &= \frac{\sqrt{x}}{10} + 400 \\ C &= \frac{\sqrt{-5p + 1000}}{10} + 400 \\ C &= \frac{\sqrt{1000 - 5p}}{10} + 400, \qquad 0 \le p \le 200 \end{aligned}$$

Thus,

$$C(p) = \frac{\sqrt{1000 - 5p}}{10} + 400, \qquad 0 \le p \le 200$$

[1] Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 258, #70.