Precalculus, Section 4.6, #44
The Graph of a Rational Function

Solve each inequality algebraically.[1]

$$\frac{5}{x-3} > \frac{3}{x+1}$$

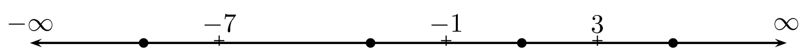
Our first task is to compare the expression to 0. A little algebra gives us

$$\begin{aligned} \frac{5}{x-3} &> \frac{3}{x+1} \\ \frac{5}{x-3} - \frac{3}{x+1} &> \frac{3}{x+1} - \frac{3}{x+1} \\ \frac{5}{x-3} - \frac{3}{x+1} &> 0 \end{aligned}$$

Our second task is to write the left side as *one* fraction. Some more algebra gives

$$\begin{aligned} \frac{5}{x-3} - \frac{3}{x+1} &> 0 \\ \frac{5}{x-3} \cdot \frac{x+1}{x+1} - \frac{3}{x+1} \cdot \frac{x-3}{x-3} &> 0 \\ \frac{5(x+1)}{(x-3)(x+1)} - \frac{3(x-3)}{(x-3)(x+1)} &> 0 \\ \frac{5(x+1) - 3(x-3)}{(x-3)(x+1)} &> 0 \\ \frac{5x+5-3x+9}{(x-3)(x+1)} &> 0 \\ \frac{2x+14}{(x-3)(x+1)} &> 0 \\ \frac{2(x+7)}{(x-3)(x+1)} &> 0 \end{aligned}$$

The number that makes the rational expression zero is $x = -7$, and the numbers that make the expression undefined are $x = 3$ and $x = -1$. We put these on a number line, and then test the value of the rational expression for a number in each of the intervals.



test point	$x=-8$		$x=-2$		$x=0$		$x=4$
$x+7$	$-$	0	$+$		$+$		$+$
$x+1$	$-$		$-$	0	$+$		$+$
$x-3$	$-$		$-$		$-$	0	$+$
$\frac{2(x+7)}{(x-3)(x+1)}$	$-$	0	$+$	UD	$-$	UD	$+$

The last row of our table shows the sign (*positive* or *negative*) of our function. Since we want the function to be greater than zero, we want the intervals where the function is positive (but *not* zero).

[1] Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 242, #44.

Because the function is undefined at $x = -1$ and $x = 3$ (the denominator is zero), neither $x = 1$ nor $x = 3$ can be a solution, and there is an open circle on the number line graph.

Finally, the solution to our inequality, written in interval notation, is

$$(-7, -1) \cup (3, \infty)$$