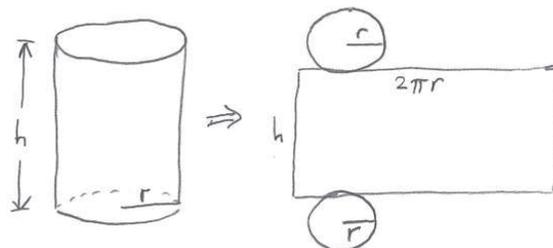


Precalculus, Section 4.5, #56
The Graph of a Rational Function

DMaterial Need to Make a Drum A steel drum in the shape of a right circular cylinder is required to have a volume of 100 cubic feet.¹

- a. Express the amount A of material required to make the drum as a function of the radius r of the cylinder.

The drum is made of two circles (top and bottom of the drum) and one rectangle (side of the drum) of material. Let h = height of the drum. See the diagram at right.



Each circle has area πr^2 , so area of the top and bottom of the drum is $2\pi r^2$.

The rectangle has height h . The base of rectangle is the circumference of the circle, so the base is $2\pi r$. This means the area of the rectangle is $2\pi r \cdot h$.

$$\text{Thus, } A(r) = 2\pi r^2 + 2\pi r h$$

Unfortunately, this is a function of the variable r and h . We need to do a substitution for h in terms of r . We are also told that the volume of the drum is 100 ft^3 and we know the volume of the drum is given by (area of base)*(height) we get $V = \pi r^2 \cdot h$ and thus

$$\begin{aligned} 100 &= \pi r^2 h \\ \frac{100}{\pi r^2} &= \frac{\pi r^2 h}{\pi r^2} \\ \frac{100}{\pi r^2} &= h \end{aligned}$$

Substituting this result into our function gives us

$$A(r) = 2\pi r^2 + 2\pi r \cdot \left(\frac{100}{\pi r^2}\right)$$

and thus the surface area of the drum as a function of the radius is given by

$$A(r) = 2\pi r^2 + \frac{200}{r}$$

- b. How much material is required if the drum's radius is 3 feet?

Here, $r = 3$ and we substitute into our function from part (a) to get

$$\begin{aligned} A(3) &= 2\pi (3)^2 + \frac{200}{3} \\ &\approx 123.22 \end{aligned}$$

Thus if the volume of the drum is 100 ft^3 and the radius is 3 ft, then the amount of material is $\approx 123.22 \text{ ft}^2$.

¹Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 236, #56.

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The Graph of a Rational Function

- c. How much material is required if the drum's radius is 4 feet?

Here, $r = 4$ and we substitute into our function from part (a) to get

$$\begin{aligned} A(4) &= 2\pi(4)^2 + \frac{200}{4} \\ &\approx 150.53 \end{aligned}$$

Thus if the volume of the drum is 100 ft^3 and the radius is 4 ft, then the amount of material is $\approx 150.53 \text{ ft}^2$.

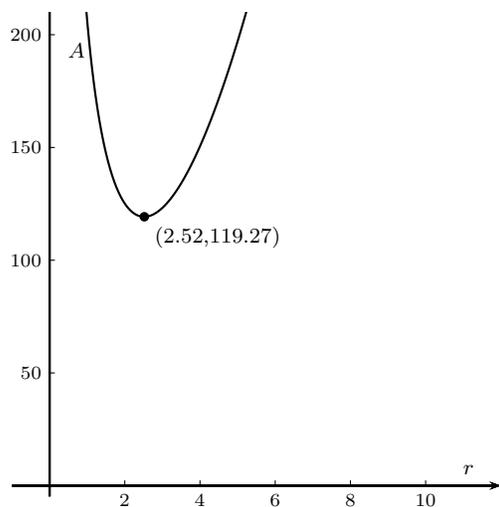
- d. How much material is required if the drum's radius is 5 feet?

Here, $r = 5$ and we substitute into our function from part (a) to get

$$\begin{aligned} A(5) &= 2\pi(5)^2 + \frac{200}{5} \\ &\approx 197.08 \end{aligned}$$

Thus if the volume of the drum is 100 ft^3 and the radius is 5 ft, then the amount of material is $\approx 197.08 \text{ ft}^2$.

- e. Graph $A = A(r)$. For what value of r is A smallest?



Using the `calc:minimum` function on the TI-84 (or whatever graphing calculator is available), we find a minimum area of $\approx 119.27 \text{ ft}^2$ occurs when the radius is $\approx 2.52 \text{ ft}$.