Find the vertical, horizontal and oblique asymptotes, if any, of each rational function.[1]

$$R(x) = \frac{8x^2 + 26x - 7}{4x - 1}$$

Let's start by writing the numerator and denominator of our function in factored form, if possible. We get

$$R(x) = \frac{(4x-1)(2x+7)}{4x-1}$$

We find vertical asymptotes (or perhaps a "hole" in the graph) determining the values of the input for which the denominator is zero. We solve

$$\begin{aligned} 4x - 1 &= 0 \\ 4x &= 1 \\ x &= \frac{1}{4} \end{aligned}$$

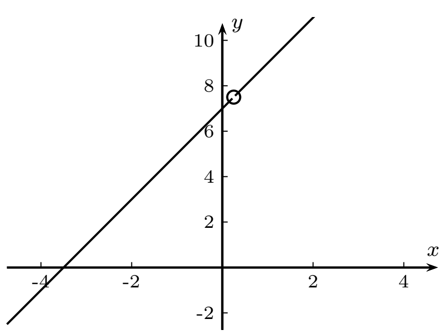
We also notice that the factor $4x-1$ apprears in both the numerator and denominator. This mean the graph will have a "hole" at $x = \frac{1}{4}$ and *not* a vertical asymptote.

Since our function $R(x) = \frac{8x^2+26x-7}{4x-1}$ has a common factor in the numerator and denominator, we can divide out that factor to get

$$\begin{aligned} R(x) &= \frac{8x^2 + 26x - 7}{4x - 1} \\ &= \frac{(4x-1)(2x+7)}{4x-1} \\ &= 2x + 7 \end{aligned}$$

for any values as long as $x \neq \frac{1}{4}$. This means that the graph of $R(x)$ is the same as the graph of $y = 2x + 7$, except for a hole at $x = \frac{1}{4}$.

The function's graph is shown below.



[1] Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 225, #52.

Find the vertical, horizontal and oblique asymptotes, if any, of each rational function.[2]

$$F(x) = \frac{x^4 - 16}{x^2 - 2x}$$

Let's start by writing the numerator and denominator of our function in factored form, if possible. Using difference of squares in the numerator twice, we get

$$F(x) = \frac{(x+2)(x-2)\left(x^2+4\right)}{x(x-2)}$$

There is a factor of x in the denominator with no common factor in the numerator. This means there will be a vertical asymptote at $x = 0$.

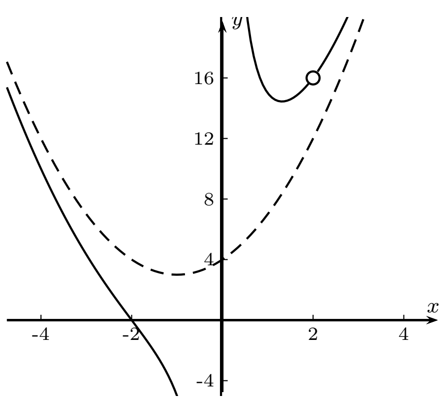
There is a factor of $x - 2$ in the denominator with a common factor in the numerator. This means there will be a "hole" in the graph at $x = 2$ and, except for that hole, the graph of $F(x)$ will be the same as the graph of $y = \frac{(x+2)\left(x^2+4\right)}{x} = \frac{x^3+2x^2+4x+8}{x}$.

To find the other possible asymptotes, we complete the long division, and examine the quotient.

$$\begin{array}{r|l}
 & x^2 + 2x + 4 \\ \hline
x & x^3 + 2x^2 + 4x + 8 \\
 & -(x^3) \\ \hline
 & 2x^2 + 4x + 8 \\
 & -(2x^2) \\ \hline
 & 4x + 8 \\
 & -(4x) \\ \hline
 & 8
\end{array}$$

The other asymptote is $y = x^2 + 2x + 4$; note that this asymptote is actually a parabola!

The function's graph is shown below.



[2]Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 225, #54.