For any function, there are three potential problems with the domain:

a. division by zero

We must exclude from the domain any values or intervals of values of the input (independent variable) that result in division by zero.

b. even roots of negative numbers

We must exclude from the domain any values or intervals of values of the input (independent variable) that result in even (square root, fourth root, etc.) roots of negative numbers.

c. values that do not make sense in an applied, real-world, problem

We must exclude from the domain any values any values or intervals of values of the input (independent variable) that do not make sense in a given applied problem.

Let's do a couple of examples.

Find the domain of each rational function.[1]

$$G(x) = \frac{x-3}{x^4+1}$$

For our function $G(x) = \frac{x-3}{x^4+1}$, there is a division operation in the function rule, so we must examine the denominator to ensure that we exclude any values of x that result in division by zero.

Let's try to find the values of x that make the denominator zero by solving

$$\begin{aligned} x^4 + 1 &= 0 \\ x^4 &= -1 \end{aligned}$$

This last equation has no solution in the real numbers (we do not allow complex numbers in functions) and so there are *no* values of x which result in division by zero.

We do not need to concern ourselves with **b** or **c**, because there are no even roots in the function rule, nor is this an applied, real-world, problem.

Thus, the domain of $G(x) = \dfrac{x-3}{x^4+1}$ is all real numbers, or in interval notation, $(-\infty, \infty)$.

[1]Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 225, #22.

Find the domain of each rational function.[2]

$$F(x) = \frac{-2\left(x^2-4\right)}{3\left(x^2+4x+4\right)}$$

For our function $F(x) = \frac{-2\left(x^2-4\right)}{3\left(x^2+4x+4\right)}$, there is a division operation in the function rule, so we must examine the denominator to ensure that we exclude any values of x that result in division by zero.

Let's try to find the values of x that make the denominator zero by solving

$$\begin{aligned} 3\left(x^2+4x+4\right) &= 0 \\ 3\left(x+2\right)\left(x+2\right) &= 0 \\ 3\left(x+2\right)^2 &= 0 \end{aligned}$$

so by the Zero Product Property,

$$\begin{aligned} x+2 &= 0 \\ x &= -2 \end{aligned}$$

So $x=-2$ would result in division by zero; we must exclude $x=-2$ from the domain. Any other value of x is just great. Since there are no even roots nor is this an applied problem, we don't concern ourselves with **b** or **c**.

Thus, the domain of $F(x)$ can be written as $\{x|x \neq -2\}$ or, in interval notation, $(-\infty,-2)\bigcup(-2,\infty)$.

[2] Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 225, #24.