Precalculus, Section 4.3, #28
Complex Zeros; Fundamental Theorem of Algebra

Use the given zero to find the remaining zeros of each function[1]

$$f(x) = x^4 - 7x^3 + 14x^2 - 38x - 60; \quad \text{zero: } 1 + 3i$$

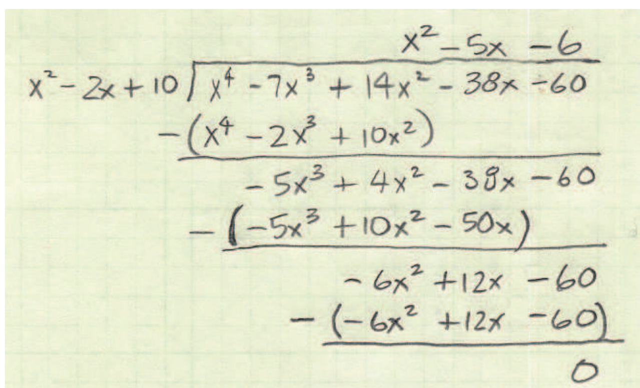
We are given the zero $1+3i$. From the Conjugate Pairs Theorem, we know that $1-3i$ must also be a zero. From the Fundamental Theorem of Algebra, we know that our function has factors

$$(x - (1+3i)) \quad \text{and} \quad (x - (1-3i))$$

If we multiply these two factors together, the product will also be a factor of $f(x)$. (This is the same as saying 2 and 3 are both factors of 12, so $2 \cdot 3 = 6$ is also a factor of 12.)

$$\begin{aligned}
(x - (1+3i))(x - (1-3i)) &= (x - 1 - 3i)(x - 1 + 3i) \\
&= x^2 - x + 3xi - x + 1 - 3i - 3xi + 3i - (3i)^2 \\
&= x^2 - 2x + 1 - (-9) \\
&= x^2 - 2x + 10
\end{aligned}$$

Now we will divide our function by this factor to find the other factor. (This is the same as saying we know 6 is a factor of 12, so we can divide $12 \div 6 = 2$ to find the other factor of 2.)



This division tells us

$$x^4 - 7x^3 + 14x^2 - 38x - 60 = \left(x^2 - 5x - 6\right)\left(x^2 - 2x + 10\right)$$

and we factor to get

$$= (x-6)(x+1)\left(x^2 - 2x + 10\right)$$

To find the zeros, we solve $f(x) = 0$, so

$$\begin{aligned}
x^4 - 7x^3 + 14x^2 - 38x - 60 &= 0 \\
(x-6)(x+1)\left(x^2 - 2x + 10\right) &= 0
\end{aligned}$$

and from Zero Product Property, we have

$$x = 6, \quad x = -1, \quad x = 1+3i, \text{ and } x = 1-3i$$

Thus the zeros of $f(x) = x^4 - 7x^3 + 14x^2 - 38x - 60$ are 6, -1, $1+3i$, and $1-3i$.

[1] Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 215, #28.