Analyze the polynomial function by following Steps 1 through 8 on page 190.[1]

$$f(x) = x^2(x-3)(x+4)$$

Step 1 *Determine the end behavior of the graph of the function.*

If we were to multiply out the function rule, we'd get

$$\begin{aligned} f(x) &= x^2(x-3)(x+4) \\ &= x^2(x-3)(x+4) \\ &= x^4 + \cdots \end{aligned}$$

so the leading term x^4 has even degree and a positive coefficient of 1. The graph of f behaves like $y = x^4$. Another way of expressing this is as $x \to \infty$, $f(x) \to \infty$ and as $x \to -\infty$, $f(x) \to \infty$.

Step 2 *Find the x- and y-intercepts of the graph of the function.*

To find the y-intercept(s), we substitute $x = 0$:

$$\begin{aligned} f(x) &= x^2(x-3)(x+4) \\ f(0) &= 0^2 * (0-3) * (0+4) \\ f(0) &= 0 \end{aligned}$$

thus the y-intercept is the point (0,0).

To find the x-intercept(s), we substitute $f(x) = 0$:

$$\begin{aligned} f(x) &= x^2(x-3)(x+4) \\ 0 &= x^2(x-3)(x+4) \end{aligned}$$

from the zero product property,

$$x^2 = 0,\ x-3 = 0,\ \text{or } x+4 = 0$$

so

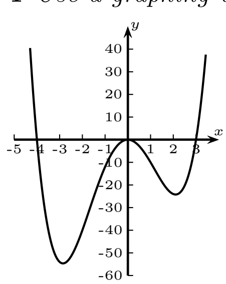
$$x = 0,\ x = 3,\ \text{or } x = -4$$

thus the x-intercepts are the points (0,0), (3,0), and (−4,0).

Step 3 *Determine the zeros of the function and their multiplicity. Use this information to determine whether the graph crosses or touches the x-axis at each x-intercept.*

From the factored form of our function, $f(x) = x^2(x-3)(x+4)$, we can see that the zeros are -4, 0, and 3. The zero -4 has multiplicity 1, so the graph of f crosses the x-axis at $x = -4$. The zero 0 has multiplicity 2, so the graph of f is tangent to the x-axis at $x = 0$ (some textbooks just say the graph touches the x-axis at $x = 0$.) The zero 3 has multiplicity 1, so the graph of f crosses the x-axis at $x = 3$.

Step 4 *Use a graphing utility to graph the function.*

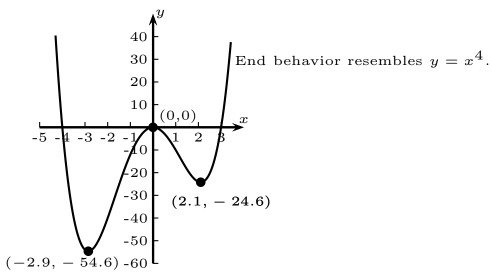


[1] Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 195, #78.

Step 5 *Approximate the turning points of the graph.*

Using the `calc:minimum` function on the TI-84 (or whatever graphing calculator is available), we find a turning point occurs at $x \approx -2.9$, at $x = 0$, and at $x \approx 2.1$.

Step 6 *Use the information in Steps 1 through 5 to draw a complete graph of the function by hand.*



Step 7 *Find the domain and range of the function.*

Because the function is a polynomial function, we know the domain is all real numbers, *i.e.*, $(-\infty, \infty)$. From the graph, the range of the function is approximately $[-54.6, \infty)$.

Step 8 *Use the graph to determine where the function is increasing and where it is decreasing.*

Using our work from Step 5 and the graph from Step 6, we can see that the function is decreasing on $(-\infty, -2.9)$, increasing on $(-2.9, 0)$, decreasing on $(0, 2.1)$, and increasing on $(2.1, \infty)$.

(Note that these are intervals of x values. This statement tells us for which values in the domain the function values are increasing or decreasing. If we need to know the y-values, we will use the function to compute those values, just as we did when finding local extrema.)