Business *The daily revenue R achieved by selling x boxes of candy is figured to be $R(x) = 9.5x - 0.04x^2$. The daily cost C of selling x boxes of candy is given by $C(x) = 1.25x + 250$.*[1]

a. *How many boxes of candy must the firm sell to maximize revenue? What is the maximum revenue?*

Note that the revenue is a quadratic function

$$R(x) = -0.04x^2 + 9.5x$$

with $a = -0.04$, $b = 9.5$, and $c = 0$, so the axis of symmetry is

$$x = -\frac{b}{2a} = -\frac{9.5}{2 \cdot -0.04} = 118.75$$

The quadratic coefficient is -0.04, so the parabola opens downward and the revenue has a maximum. The maximum revenue is given by

$$R(118.75) = -0.04\,(118.75)^2 + 9.5 \cdot 118.75 = 564.06$$

Thus the maximum revenue of \$564.06 occurs when we sell 118.75 boxes of candy.

b. *Profit is given as $P(x) = R(x) - C(x)$. What is the profit function?*

$$\begin{aligned} P(x) &= R(x) - C(x) \\ P(x) &= \left(9.5x - 0.04x^2\right) - (1.25x + 250) \\ &= 9.5x - 0.04x^2 - 1.25x - 250 \end{aligned}$$

Thus

$$P(x) = -0.04x^2 + 8.25x - 250$$

c. *How many boxes of candy must the firm sell to maximize profit? What is the maximum profit?*

Note that the profit is a quadratic function

$$P(x) = -0.04x^2 + 8.25x - 250$$

with $a = -0.04$, $b = 8.25$, and $c = -250$, so the axis of symmetry is

$$x = -\frac{b}{2a} = -\frac{8.25}{2 \cdot -0.04} = 103.125$$

The quadratic coefficient is -0.04, so the parabola opens downward and the profit has a maximum. The maximum profit is given by

$$P(103.125) = -0.04\,(1103.125)^2 + 8.25 \cdot 103.125 - 250 = 175.39$$

Thus the maximum profit of \$175.39 occurs when we sell 103.125 boxes of candy.

d. *Provide a reasonable explanation as to why the answers found in (a) and (c) differ. Explain why a quadratic function is a reasonable model for revenue.*

The maximums for revenue and profit occur at different values of x because as revenue is increasing, the costs of production are also increasing, and eventually the cost to produce the next item exceeds the revenue gained from selling the next item. Thus profit is maximized before revenue is maximized.

A quadratic function is a reasonable model for revenue because *revenue* $=$ *price* $\cdot$ *quantity*. The price will be a function of the number of boxes of candy sold (this is the law of supply and demand) and thus revenue is the product of two functions of x.

[1] Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 158, #92.