Precalculus, Section 1.4, #66
Lines

Find an equation for the line with the given properties[1]*. Express your answer using either the general form or the slope-intercept form of the equation of a line, whichever you prefer.*

Perpendicular to the line $y = 2x - 3$; containing the point $(1, -2)$

To write the equation of a line, we need the slope of the line and a point on the line.

Since our desired line is perpendicular to the given line $y = 2x - 3$, we know the slope of our desired line is $-\frac{1}{2}$. (The slope of perpendicular lines are opposite reciprocals.)

Let's start to write the equation of our desired line. Since the slope is $-\frac{1}{2}$, we have

$$y = -\frac{1}{2}x + b$$

We are also told that our desired line contains the point $(1, -2)$. Since this point is on our desired line, its x- and y-values must make our desired equation true. We substitute $= 1$ and $y = -2$ into our (unfinished) equation and solve for b.

$$\begin{aligned} y &= -\frac{1}{2}x + b \\ -2 &= -\frac{1}{2} \cdot (1) + b \\ -2 &= -\frac{1}{2} + b \\ -2 + \frac{1}{2} &= -\frac{1}{2} + b + \frac{1}{2} \\ -\frac{4}{2} + \frac{1}{2} &= b \\ -\frac{3}{2} &= b \end{aligned}$$

Thus the slope-intercept form of the equation perpendicular to $y = 2x - 3$; containing the point $(1, -2)$ is

$$y = -\frac{1}{2}x - \frac{3}{2}$$

To find the general form, $Ax + By = C$, we just do the algebra to transform our slope-intercept form into the general form

$$\begin{aligned} y &= -\frac{1}{2}x - \frac{3}{2} \\ 2 \cdot y &= 2\left(-\frac{1}{2}x - \frac{3}{2}\right) \\ 2y &= -x - 3 \\ 2y + x &= -x - 3 + x \\ 2y + x &= -3 \end{aligned}$$

Thus the general form of the equation perpendicular to $y = 2x - 3$; containing the point $(1, -2)$ is

$$2y + x = -3$$

---

[1] Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 41, #66.